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Class XII, Applied Mathematics

CHAPTER SUMMARY- MATRICS AND DETERMINANTS

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TOPIC

- Define matrix
- Different kinds of matrices
- The size / order of matrices
- Equality of two matrices
- Transpose of a matrix
- Symmetric and skew symmetric matrix
- Operations like addition & subtraction on matrices of same order
- Multiplication of two matrices of appropriate order
- Multiplication of a scalar with matrix
- Inverse of a square matrix
- Elementary row operations and use it to find the inverse of a matrix
- Properties of inverse of matrices
- Determinant of a square matrix
- Elementary properties of determinants
- Solve the system of simultaneous equations using i) Cramer's Rule ii) Inverse of coefficient matrix iii) Row reduction method
- Real life problems into a system of simultaneous linear equations and solve the equations.
- Simple applications of matrices and determinants in different areas of mathematics.
- Real life applications particularly for Leontief input output model for two variables in economics

DEFINITIONS

Types of matrices:

Row matrix: A matrix is said to be a row matrix if it has only one row. E.g. [a b c]

Column matrix: A matrix is said to be a column matrix if it has only one column. E.g. $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

Square matrix: A matrix in which the number of rows is equal to the number of columns. E.g. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Diagonal matrix:

$A\ square\ matrix\ A = [a_{ij}]\ is\ said\ to\ be\ a\ diagonal\ matrix$

$if\ its\ all\ non\ diagonal\ elements\ are\ zero.$ E.g. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Scalar Matrix: A diagonal matrix is said to be a scalar matrix, if its diagonal elements are equal.

E.g. $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Identity Matrix: A square matrix in which elements in the diagonal are all 1 and rest are all zeroes. E.g. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Zero matrix: A matrix is said to be a zero matrix if all its elements are zeroes.

Symmetric matrix: A square matrix is said to be symmetric if $A^T = A$

Skew symmetric matrix: A square matrix is said to be symmetric if $A^T = -A$

Singular matrix: A square matrix A is said to be a singular matrix if $|A| = 0$.

IMPORTANT POINTS/PROPERTIES

1. Two matrices are equal if they are of the same order and their corresponding elements are equal
2. For Addition/subtraction of two matrices, their order should be same.
3. If A is a matrix of order m x n, B is a matrix of order n x p then Product AB will be a matrix of order m x p.
4. Matrix multiplication is not commutative. $AB \neq BA$
5. $(A+B)^T = A^T + B^T$
6. $(AB)^T = B^T A^T$
7. $(AB)^{-1} = B^{-1}A^{-1}$
8. For any square matrix A, A = P +Q, P is a symmetric and Q is a skew symmetric matrix and
$$P = \frac{1}{2}(A + A^T)\ and\ Q = \frac{1}{2}(A - A^T)$$
9. Inverse of a matrix: If A is a square matrix, then A is called invertible if and only if there exists a square matrix B such that $A.B = I = B.A$.
Note: $(A^{-1})^{-1} = A$
10. Elementary row wise transformation:
To find A^{-1} using elementary row operations, write $A = IA$ and apply a sequence of row operations on $A = IA$ till we get $I = BA$. The matrix B will be the inverse of A. ($where\ I$ is represents identity matrix)
Row operations: (i) $R_i \leftrightarrow R_j$
(ii) $R_i \to kR_i, k \neq 0$
(iii) $R_i \to R_i + kR_j$
Note: In case after applying one or more elementary row operations on $A = IA$, if we obtain all zeros in one or more rows of the matrix on LHS, then A^{-1} does not exist.
11. $A^{-1} = \frac{adjA}{|A|}$
12. Minor of an element a_{ij} $of\ the\ determinant\ of\ matrix\ A\ is\ the$ determinant obtained by deleting i^{th} row and j^{th} column, and it is denoted by M_{ij}
13. Co-factor of an element $a_{ij}\ is\ given\ by\ A_{ij} = (-1)^{i+j}M_{ij}$
14. The sum of the products of elements of any row (or column) of a determinant with their corresponding cofactors is equal to the value of the determinant.
$$|A| = a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{13}$$
15. $A.adjA = |A|.I$
16. If A$(x_1, y_1), B\ (x_2, y_2)\ and\ C(x_3, y_3), then\ a$rea of triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

If the points are collinear, then area = 0

17. $|A.adjA| = |A|^n, where\ n\ is\ the\ order\ of\ the\ matrix\ A.$

18. $|adjA| = |A|^{n-1}$, where n is the order of matrix A.

19. $|kA| = k^n|A|$, where k is a constant and n is the order of matrix A.

20. $|AB| = |A||B|$

21. $|A| = |A^T|$

22. If any two row or two columns of a determinant are interchanged, then the sign of the value of determinant changes,

23. If all the elements of a row or a column of a determinant are zeroes **OR**
Any two rows or columns of a determinant are identical **OR**
If any two rows or columns of determinant are proportional, then the value of the determinant is zero.

24. Multiplying a determinant by k means multiplying the elements of only one row (or one column) by k

25. If to each element of a row (or a column) of a determinant the equal multiples of corresponding elements of other rows(columns) are added, then value of determinant remains the same.

26. If elements of a row (or a column) in a determinant can be expressed as the sum of two or more elements, then the given determinant can be expressed as the sum of two or more determinants.

27. Leontief input -output model:
$(I - A)X = D$, where the matrix A is called input-output coefficient matrix or technology matrix, X is the output matrix and D is the demand matrix.
Note: $The\ matrix\ (I - A) is\ called\ Leontief\ matrix.$

28. Hawkins-Simon conditions:
The system is viable if
   (1) $|I - A| > 0$
   (2) $(2)\ The\ diagonal\ elements\ of\ the\ Leontief\ matrix\ (I - A)\ \ should\ all\ be\ positive.$

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