



# INDIAN SCHOOL AL WADI AL KABIR

**Class XII**, Mathematics **Worksheet 2- Continuity and Differentiability**

**28-06-20**

## Short answer type (2 marks)

**Q.1.** Differentiate  $\sin^2 x$  with respect to  $e^{\cos x}$ .

**Q.2** Find the value of  $k$  so that the function is continuous at the given value:

$$f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x, & x > \pi \end{cases}$$

**Q.3** Find  $\frac{dy}{dx}$ , if  $\sin^2 x + \cos xy = \pi$

**Q.4** Find  $\frac{dy}{dx}$ , if  $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}$

**Q.5** Find  $\frac{dy}{dx}$ , if  $y = \tan^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right)$

**Q.6** Find  $\frac{dy}{dx}$ , if  $x^3 + y^3 + xy = 10$

**Q.7** Find the values of  $a$  and  $b$  if the function  $f$  defined by

$$f(x) = \begin{cases} x^3 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases} \text{ is differentiable at } x = 1$$

**Q.8** Verify Rolle's theorem for the function  $f(x) = x^2 + 2x + 8, x \in [-4, 2]$

**Q.9** Verify mean value theorem for the function  $f(x) = 2\sin x + \sin 2x, x \in [0, \pi]$

**Q.10** If  $x = e^{\cos 2t}$  and  $y = e^{\sin 2t}$ , prove that  $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$

## Long answer type (4 Marks)

**Q.11** If  $f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx} - 1}{x}, & x > 0 \end{cases}$  is continuous at  $x = 0$ , find the values of  $a$  and  $b$ .

Q.12	If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ , then prove: $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$
Q.13	If $x = a\left(\cos\theta + \log \tan \frac{\theta}{2}\right)$ , $y = a \sin\theta$ , then find $\frac{dy}{dx}$
Q.14	If $x^p y^q = (x+y)^{p+q}$ , then prove: $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$
Q.15	If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ , find $f'\left(\frac{\pi}{2}\right)$
Q.16	If $x = a \sin pt$ , $y = b \cos pt$ , then prove $(a^2 - x^2)y \frac{d^2y}{dx^2} + b^2 = 0$
Q.17	If $y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$ , then find $\frac{dy}{dx}$ .
Q.18	If $x \cos(a+y) = \cos y$ , then prove: $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ and $\sin a \cdot \frac{d^2y}{dx^2} + \sin 2(a+y) \frac{dy}{dx} = 0$
Q.19	If $y = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ , then find $\frac{dy}{dx}$
Q.20	If $y = (\tan^{-1}x)^2$ then prove: $(1+x^2)^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 2$

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<b>Answers</b>	1	$-2\cos x \cdot e^{-\cos x}$	2	$-\frac{2}{\pi}$	3.	$\frac{\sin 2x - y \sin xy}{x \sin xy}$	4	0
	5	$\frac{3}{1+x^2}$	6	$-\frac{(3x^2+y)}{(3y^2+x)}$	7	$a=4, b=6$	8	$-1 \in (-4, 2)$
	9	$\frac{\pi}{3} \in (0, \pi)$	11	$a=-1, b=4$	13	$\tan\theta$	15	1
	17	$x^2(\cos x)^x [3 - x^2 \tan x + x \log \cos x] + \frac{1}{2\sqrt{x-x^2}}$					19	$\sqrt{a^2 - x^2}$