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Secondary School Examination - 2020

Marking Scheme- MATHEMATICS BASIC

Subject Code : 241 Paper Code: 430/3/1,2,3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.**Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best effortsin this process.**

2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-X, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**

3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).

11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 430/3/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

Select the correct choice.

1. What is the largest number that divides 245 and 1029, leaving remainder 5 in each?

(a) 15 **(b) 16** **(c) 9** **(d) 5**

Sol. (b) 16 1

---

2. Consider the following distribution:

Classes:	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency:	10	15	12	20	9

The sum of lower limits of the median class and the modal class is

(a) 15 **(b) 25** **(c) 30** **(d) 35**

Sol. (b) 25 1

---

3. If the two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is:

(a) 3 cm **(b) $\frac{3\sqrt{3}}{2}$ cm** **(c) $3\sqrt{3}$ cm** **(d) 6 cm**

Sol. (c) $3\sqrt{3}$ cm 1

---

4. The simplest form of $\frac{1095}{1168}$ is

(a) $\frac{17}{26}$ **(b) $\frac{25}{26}$** **(c) $\frac{13}{16}$** **(d) $\frac{15}{16}$**

Sol. (d) $\frac{15}{16}$ 1

---

5. One card is drawn at random from a well – shuffled deck of 52 cards. What is the probability of getting a Jack?

(a) $\frac{3}{26}$ **(b) $\frac{1}{52}$** **(c) $\frac{1}{13}$** **(d) $\frac{3}{52}$**

Sol. (c) $\frac{1}{13}$ 1

---

6. If one zero of the quadratic polynomial, (k – 1) x^2 + kx + 1 is –4 then the value of k is

(a) $-\frac{5}{4}$ **(b)** $\frac{5}{4}$ **(c)** $-\frac{4}{3}$ **(d)** $\frac{4}{3}$

Sol. (b) $\frac{5}{4}$ 1

---

7. Which of the following rational numbers is expressible as a terminating decimal?

(a) $\frac{124}{165}$ **(b)** $\frac{131}{30}$ **(c)** $\frac{2027}{625}$ **(d)** $\frac{1625}{462}$

Sol. (c) $\frac{2027}{625}$ 1

---

8. If α and β are the zeros of ($2x^2 + 5x - 9$), then the value of $\alpha\beta$ is

(a) $-\frac{5}{2}$ **(b)** $\frac{5}{2}$ **(c)** $-\frac{9}{2}$ **(d)** $\frac{9}{2}$

Sol. (c) $\frac{-9}{2}$ 1

---

9. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(a) $7+\sqrt{5}$ **(b) 5** **(c) 10** **(d) 12**

Sol. (d) 12 1

---

10. If P(–1, 1) is the midpoint of the line segment joining A(–3, b) and B(1, b + 4), then b is equal to

(a) 1 **(b) –1** **(c) 2** **(d) 0**

Sol. (b) –1 1

In Question numbers 11 to 15, fill in the blanks:

11. Distance between (a, –b) and (a, b) is ________.

Sol. 2b units 1

---

12. The value of k for which system of equations x + 2y = 3 and 5x + ky = 7 has no solution is ________.

Sol. k = 10 1

---

13. The value of ($\cos^2 45^\circ + \cot^2 45^\circ$) is ________.

Sol. $\frac{3}{2}$ 1

---

14. The value of (tan 27º – cot 63º) is ________.

Sol. 0 1

---

15. If ratio of the corresponding sides of two similar triangles is 2:3, then ratio of their perimeters is _________.

Sol. 2 : 3 1

Answer the following questions, Question numbers 16 to 20.

16. If $\sec\theta = \dfrac{25}{7}$, then find the value of $\cot\theta$.

Sol. $\tan\theta = \dfrac{24}{7} \Rightarrow \cot\theta = \dfrac{7}{24}$ $\frac{1}{2}+\frac{1}{2}$

OR

If $3\tan\theta = 4$, then find the value of $\left(\dfrac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}\right)$

Sol. Given expression $= \dfrac{3\times\frac{4}{3}+2}{3\times\frac{4}{3}-2} = 3$ $\frac{1}{2}+\frac{1}{2}$

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17. The perimeter of a sector of a circle of radius 14 cm is 68 cm. Find the area of the sector.

Sol. $l = 68 - 28 = 40$ cm $\frac{1}{2}$

$A = 280 \text{ cm}^2$ $\frac{1}{2}$

OR

The circumference of a circle is 39.6 cm. Find its area.

Sol. $r = \dfrac{39.6}{2\pi}$ $\frac{1}{2}$

$A = \dfrac{392.04}{\pi}$ or 124.74 cm^2 $\frac{1}{2}$

---

18. A letter of English alphabet is chosen at random. Determine the probability that chosen letter is a consonant.

Sol. No. of consonents = 21 $\frac{1}{2}$

$\therefore P = \dfrac{21}{26}$ $\frac{1}{2}$

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19. **In Fig. 1, D and E are points on sides AB and AC respectively of a ΔABC such that DE || BC. If AD = 3.6 cm, AB = 10 cm and AE =4.5 cm, find EC and AC.**

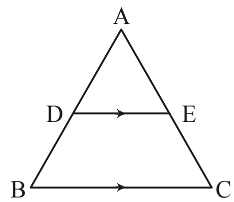

Fig. 1

Sol. EC = 8 cm $\frac{1}{2}$

AC = 12.5 cm $\frac{1}{2}$

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20. **If 3y – 1, 3y + 5 and 5y + 1 are three consecutive terms of an A.P., then find the value of y.**

Sol. $2(3y + 5) = 3y - 1 + 5y + 1$ $\frac{1}{2}$

$y = 5$ $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. **A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is**

(i) red or white

(ii) not a white ball

Sol. Total no. of balls = 20

(i) P(ball is red or white) = $\frac{13}{20}$ 1

(ii) P(Not a white ball) = $\frac{12}{20}$ or $\frac{3}{5}$ 1

---

22. **Two dice are thrown at the same time. Find the probability of getting different numbers on the two dice.**

Sol. Total number of outcomes = 36 $\frac{1}{2}$

Favourable numbers of outcomes = 30 $\frac{1}{2}$

Probability = $\frac{30}{36}$ or $\frac{5}{6}$ 1

$\begin{pmatrix}\text{Both numbers} \\ \text{are different}\end{pmatrix}$

OR

Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is more than 9.

Sol. Favourable outcomes (5, 5), (4, 6), (6, 4), (6, 5), (5, 6), (6, 6)

Total number of outcomes = 36 $\frac{1}{2}$

Number of favourable outcomes = 6 $\frac{1}{2}$

Required probability = $\frac{6}{36}$ or $\frac{1}{6}$ 1

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23. In Fig. 2, a circle is inscribed in a ΔABC, touching BC, CA and AB at P, Q and R respectively. If AB = 10 cm, AQ = 7 cm and CQ = 5 cm then find the length of BC.

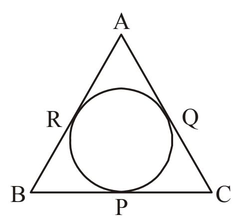

Fig. 2

Sol. AQ = AR = 7 cm $\frac{1}{2}$

BR = AB – AR = 10 – 7 = 3 cm $\frac{1}{2}$

BC = BP + PC

= BR + CQ $\frac{1}{2}$

= 3 + 5 = 8 cm $\frac{1}{2}$

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24. Prove that: $\sqrt{\sec^2\theta + \text{cosec}^2\theta}$ = tan θ + cot θ

Sol. LHS = $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$ 1

$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta}$

$= \sqrt{(\tan\theta + \cot\theta)^2}$ $\frac{1}{2}$

$= \tan\theta + \cot\theta = \text{RHS}$ $\frac{1}{2}$

OR

Prove that: $\frac{\sin\theta}{1-\cos\theta} = (\text{cosec}\ \theta + \cot\theta)$

Sol. LHS $= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$ 1

$= \frac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta}$ $\frac{1}{2}$

$= \frac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= \text{cosec}\ \theta + \cot\theta = \text{RHS}$ $\frac{1}{2}$

---

25. Three cubes each of volume 216 cm^3 are joined end to end to form a cuboid. Find the total surface area of resulting cuboid.

Sol. $a^3 = 216\ cm^3$

$a = 6\ cm$ 1

TSA of cuboid $= 5a^2 + 4a^2 + 5a^2$

$= 14a^2$ $\frac{1}{2}$

$= 504\ cm^2$ $\frac{1}{2}$

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26. Find the values of p for which the quadratic equation $x^2 - 2px + 1 = 0$ has no real roots.

Sol. For no real roots

$D < 0$

$(-2p)^2 - 4 \times 1 \times 1 < 0$ 1

$p^2 - 1 < 0$ $\frac{1}{2}$

$-1 < p < 1$ $\frac{1}{2}$

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. If 1 and –2 are the zeroes of the polynomial $(x^3 - 4x^2 - 7x + 10)$, find its third zero.

Sol. The two factors of polynomials are $(x - 1)$, $(x + 2)$ $\frac{1}{2}$

$(x - 1)(x + 2) = x^2 + x - 2$ $\frac{1}{2}$

$$\frac{x^3 - 4x^2 - 7x + 10}{x^2 + x - 2} = (x - 5)$$ $1\frac{1}{2}$

Third zero = 5 $\frac{1}{2}$

---

28. Draw a circle of radius 3 cm. From a point 7 cm away from its centre, construct a pair of tangents to the circle.

Sol. Drawing a circle of radius 3 cm, marking Centre 0 and taking a point P such that OP = 7 cm 1

Constructing two tangents 2

OR

Draw a line segment of 8 cm and divide it in the ratio 3 : 4.

Sol. Drawing a line segment of 8 cm 1

Dividing it in the ratio 3 : 4 2

---

29. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm^2. If the same wire is bent into the form of a circle, what will be the radius of the circle?

Sol. Let 'a' be the side of the equilateral triangle

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 121\sqrt{3}$$ 1

$\Rightarrow$ a = 22 cm $\frac{1}{2}$

Perimeter of triangle = 3a = 66 cm $\frac{1}{2}$

Hence, $2\pi r = 66$ cm $\frac{1}{2}$

$$r = \frac{33}{\pi} \text{ cm or } \frac{21}{2} \text{ cm}$$ $\frac{1}{2}$

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30. Prove that $\dfrac{\cos\theta}{(1-\tan\theta)}+\dfrac{\sin\theta}{(1-\cot\theta)}$ = $(\cos\theta + \sin\theta)$

Sol. LHS $= \dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin\theta}{1-\cot\theta}$

$= \dfrac{\cos^2\theta}{\cos\theta-\sin\theta}+\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}$ 1

$= \dfrac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$ 1

$= \cos\theta + \sin\theta$ = RHS 1

OR

Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

Sol. $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \text{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$ $\frac{1}{2}+\frac{1}{2}$

$= \sin^2\theta + 1 + \cot^2\theta + 2 + \cos^2\theta + 1 + \tan^2\theta + 2$ $\frac{1}{2}+\frac{1}{2}$

$= 7 + \tan^2\theta + \cot^2\theta$ 1

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31. If $\sqrt{2}$ is given as an irrational number, then prove that $(7 - 2\sqrt{2})$ is an irrational number.

Sol. Let $7-2\sqrt{2} = m$, where m is a rational number $\frac{1}{2}$

$\sqrt{2} = \dfrac{7-m}{2}$ 1

Irrational = Rational 1

$\Rightarrow$ LHS $\neq$ RHS

It means out assumption is wrong.

Hence, $7-2\sqrt{2}$ is irrational $\frac{1}{2}$

OR

Find HCF of 44, 96 and 404 by prime factorization method. Hence find their LCM.

Sol. $\left.\begin{array}{l} 44 = 2^2 \times 11 \\ 96 = 2^5 \times 3 \\ 404 = 2^2 \times 101 \end{array}\right]$ $1\frac{1}{2}$

HCF $= 2^2 = 4$ $\frac{1}{2}$

LCM $= 2^5 \times 11 \times 3 \times 101$

$= 106656$ 1

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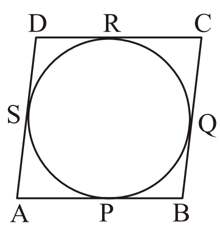
32. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. Correct figure $\frac{1}{2}$



$\left.\begin{array}{l} AP = AS \\ BP = BQ \\ CQ = CR \\ DR = DS \end{array}\right]$ Tangents from external point 1

$AB + DC = AP + PB + DR + RC$

$= AS + BQ + DS + CQ$

$= AD + BC$ 1

Since, ABCD is a llgm, $AB = DC$, $AD = BC$

$2AB = 2AD$

$AB = AD$

$\Rightarrow$ ABCD is a rhombus $\frac{1}{2}$

---

33. **In Fig. 3, arrangement of desks in a classroom is shown. Ashima, Bharti and Asha are seated at A, B and C respectively. Answer the following:**

(i) Find whether the girls are sitting in a line.

(ii) If A, B and C are collinear, find the ratio in which point B divides the line segment joining A and C.

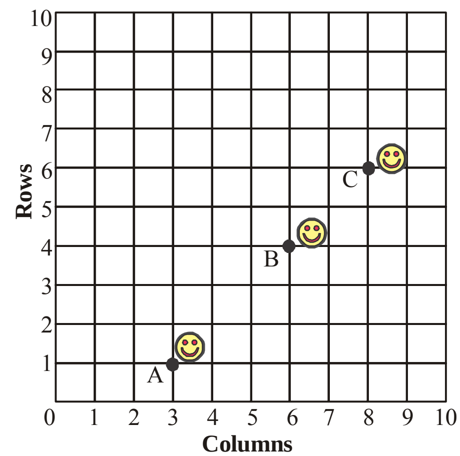


Fig. 3

Sol. Coordinates of A(3, 1)

B(6, 4)

C(8, 6) 1

(i) Area of (ΔABC) $= \frac{1}{2}[3(4-6)+6(6-1)+8(1-4)]$ $\frac{1}{2}$

$= 0$

Yes they are sitting in same line $\frac{1}{2}$

(ii) Let AB : BC = k : 1

$6 = \frac{8k+3}{k+1}$ $\frac{1}{2}$

$k = \frac{3}{2}$ or Ratio $= 3:2$ $\frac{1}{2}$

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34. **A number consists of two digits whose sum is 10. If 18 is subtracted from the number, its digit are reversed. Find the number.**

Sol. Let two digit number = 10x + y $\frac{1}{2}$

$x + y = 10$ ...(i) $\frac{1}{2}$

$10x + y - 18 = 10y + x$

$\Rightarrow$ $x - y = 2$ ...(ii) 1

On solving (i) & (ii) x = 6, y = 4 $\frac{1}{2}$

$\therefore$ Required number = 64 $\frac{1}{2}$

SECTION D

Question Nos. 35 to 40 carry 4 marks each.

35. **Some students planned a picnic. The total budget for food was ₹ 2,000 but 5 students failed to attend the picnic and thus the cost for food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food?**

Sol. Let number of students be x

Cost of food for one student = ₹ $\frac{2000}{x}$ $\frac{1}{2}$

$(x - 5)\left(\frac{2000}{x} + 20\right) = 2000$ 1

$x^2 - 5x - 500 = 0$

$(x - 25)(x + 20) = 0$ 1

$x = 25$ $\frac{1}{2}$

No. of students attended picnic = 20 $\frac{1}{2}$

Cost of food they pay = ₹ 100 $\frac{1}{2}$

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36. **The sum of first 6 terms of an A.P. is 42. The ratio of its 10th term to 30th term is 1:3. Find the first and the 13th term of the A.P.**

Sol. Here, $\frac{6}{2}(2a + 5d) = 42$

$\Rightarrow$ $2a + 5d = 14$ ...(i) 1

Also,

$\frac{a + 9d}{a + 29d} = \frac{1}{3}$ ...(ii) 1

$\Rightarrow$ $a = d$ $\frac{1}{2}$

Solving (i) and (ii), $7a = 14$ $\frac{1}{2}$

$\Rightarrow$ $a = 2$

$d = 2$ $\frac{1}{2}$

$a_{13} = a + 12d = 26$ $\frac{1}{2}$

OR

Find the sum of all odd numbers between 100 and 300.

Sol. Odd number between 100 to 300 are 1

101, 103 ... 299

299 = 101 + (n – 1)2

$\Rightarrow$ n = 100 1

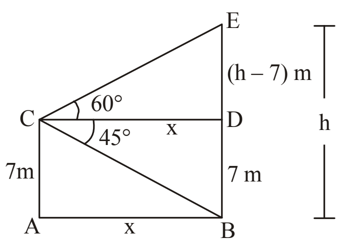
$S_n = \frac{100}{2}(101+299)$ 1

= 20,000 1

---

37. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60°, and the angle of depression of its foot is 45°. Find the height of the tower. Given that $\sqrt{3}$ = 1.732.

Sol.



Correct figure 1

$\tan 45° = \frac{7}{x}$

$\Rightarrow$ x = 7 m ...(i) 1

$\tan 60° = \frac{h-7}{x}$

$x\sqrt{3} = h - 7$ ....(ii) 1

Solving (i) and (ii), $h = 7(\sqrt{3}+1)$

= 7 × 2.732

= 19.124 m 1

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38. In a right triangle, prove that the square of the hypotenuse is equal to sum of squares of the other two sides.

Sol. For correct given, to prove, construction and figure $4\times\frac{1}{2}=2$

For correct proof 2

OR

Prove that the tangents drawn from an external point to a circle are equal in length.

Sol. For correct given, to prove, construction and figure $4\times\frac{1}{2}=2$

For correct proof 2

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39. A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to edge of the cube. Determine the volume of the remaining block.

Sol. Let r be the radius of hemisphere $\therefore\ r = \frac{21}{2}$ cm $\frac{1}{2}$

Volume of remaining block $= a^3 - \frac{2}{3}\pi r^3$

$= (21)^3 - \frac{2}{3}\pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$ 2

$= 9261\left[1 - \frac{\pi}{12}\right] \text{cm}^3$ 1

$= 6853 \text{ cm}^3$ (Approx.) $\frac{1}{2}$

OR

A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere of same radius. Find the radius of the hemisphere and total height of the toy, if the height of the cone is 3 times the radius.

Sol. Here, r = 6 cm

$\pi(6)^2 \times 15 = 12\left[\frac{1}{3}\pi r^2 \times 3r + \frac{2}{3}\pi r^3\right]$ 2

$36 \times 15 = \frac{12}{3}[3r^3 + 2r^3]$ $\frac{1}{2}$

$9 \times 15 = 5r^3$

r = 3 cm $\frac{1}{2}$

Total height = 12 cm 1

40. Find the mean of the following data:

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**
Frequency	**5**	**10**	**18**	**30**	**20**	**12**	**5**

Sol.

CI	f_i	x_i	d_i	u_i	f_iu_i
0-10	5	5	–30	–3	–15
10-20	10	15	–20	–2	–20
20-30	18	25	–10	–1	–18
30-40	30	35	0	0	0
40-50	25	45	10	1	20
50-60	12	55	20	2	24
60-70	5	65	30	3	15
Total	100				6

Correct Table 2

$$\text{mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 35 + \frac{6}{100} \times 10$$ 1

$$= \frac{356}{10} \text{ or } 35.6$$ 1

QUESTION PAPER CODE 430/3/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

Select the correct choice.

1. The simplest form of $\frac{1095}{1168}$ is

(a) $\frac{17}{26}$ (b) $\frac{25}{26}$ (c) $\frac{13}{16}$ (d) $\frac{15}{16}$

Sol. (d) $\frac{15}{16}$ 1

---

2. One card is drawn at random from a well – shuffled deck of 52 cards. What is the probability of getting a Jack?

(a) $\frac{3}{26}$ (b) $\frac{1}{52}$ (c) $\frac{1}{13}$ (d) $\frac{3}{52}$

Sol. (c) $\frac{1}{13}$ 1

---

3. Which of the following rational numbers is expressible as a terminating decimal?

(a) $\frac{124}{165}$ (b) $\frac{131}{30}$ (c) $\frac{2027}{625}$ (d) $\frac{1625}{462}$

Sol. (c) $\frac{2027}{625}$ 1

---

4. If one zero of the quadratic polynomial, $(k - 1) x^2 + kx + 1$ is -4 then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{5}{4}$ (c) $-\frac{4}{3}$ (d) $\frac{4}{3}$

Sol. (b) $\frac{5}{4}$ 1

---

5. If P(–1, 1) is the midpoint of the line segment joining A(–3, b) and B(1, b + 4), then b is equal to

(a) 1 (b) –1 (c) 2 (d) 0

Sol. (b) –1 1

---

6. Consider the following distribution:

Classes:	**0 – 5**	**5 – 10**	**10 – 15**	**15 – 20**	**20 – 25**
Frequency:	**10**	**15**	**12**	**20**	**9**

The sum of lower limits of the median class and the modal class is

(a) 15 **(b) 25** **(c) 30** **(d) 35**

Sol. (b) 25 1

---

7. What is the largest number that divides 245 and 1029, leaving remainder 5 in each?

(a) 15 **(b) 16** **(c) 9** **(d) 5**

Sol. (b) 16 1

---

8. The distance between the points A(2, –3) and B(2, 2) is

(a) 2 units **(b) 3 units** **(c) 4 units** **(d) 5 units**

Sol. (d) 5 units 1

---

9. The product of the two zeroes of the polynomial $3x^2 - 7x - 27$ is:

(a) 27 **(b) 9** **(c) –9** **(d) $\frac{7}{3}$**

Sol. (c) –9 1

---

10. If the tangents PA and PB from an external point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA equals:

(a) 50° **(b) 60°** **(c) 70°** **(d) 80°**

Sol. (a) 50° 1

In Question numbers 11 to 15, fill in the blanks:

11. The value of k for which system of equations x + 2y = 3 and 5x + ky = 7 has no solution is ________.

Sol. k = 10 1

---

12. The value of (tan 27° – cot 63°) is ________.

Sol. 0 1

---

13. If ratio of the corresponding sides of two similar triangles is 2:3, then ratio of their perimeters is _________.

Sol. 2 : 3 1

---

14. Distance between (a, –b) and (a, b) is ________.

Sol. 2b units 1

---

15. **The value of (sin 20° – cos 70°) is _______.**

Sol. 0 1

Answer the following questions, Question numbers 16 to 20.

16. **The perimeter of a sector of a circle of radius 14 cm is 68 cm. Find the area of the sector.**

Sol. l = 68 – 28 = 40 cm $\frac{1}{2}$

A = 280 cm^2 $\frac{1}{2}$

OR

The circumference of a circle is 39.6 cm. Find its area.

Sol. $r = \dfrac{39.6}{2\pi}$ $\frac{1}{2}$

$A = \dfrac{392.04}{\pi}$ or $124.74\ \text{cm}^2$ $\frac{1}{2}$

---

17. **If 3y – 1, 3y + 5 and 5y + 1 are three consecutive terms of an A.P., then find the value of y.**

Sol. 2(3y + 5) = 3y – 1 + 5y + 1 $\frac{1}{2}$

y = 5 $\frac{1}{2}$

---

18. **If sec θ = $\mathbf{\dfrac{25}{7}}$, then find the value of cot θ.**

Sol. $\tan\theta = \dfrac{24}{7} \Rightarrow \cot\theta = \dfrac{7}{24}$ $\frac{1}{2}+\frac{1}{2}$

OR

If 3 tan θ = 4, then find the value of $\left(\dfrac{\mathbf{3\sin\theta + 2\cos\theta}}{\mathbf{3\sin\theta - 2\cos\theta}}\right)$

Sol. Given expression = $\dfrac{3\times\frac{4}{3}+2}{3\times\frac{4}{3}-2} = 3$ $\frac{1}{2}+\frac{1}{2}$

---

19. **A bag contains 5 red, 4 blue and 3 green balls. A ball is drawn at random from the bag. Find the probability of getting a ball not of blue colour.**

Sol. Total No. of balls = 12 $\frac{1}{2}$

P(Not a blue ball) = $\dfrac{8}{12}$ or $\dfrac{2}{3}$ $\frac{1}{2}$

---

20. In Fig. 1, DE || BC, AD = 2.4 cm, AE = 3.2 cm and CE = 4.8 cm. Find BD

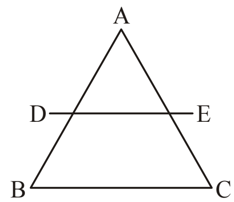


Fig. 1

Sol. $\frac{2.4}{BD} = \frac{3.2}{4.8}$ $\frac{1}{2}$

$\Rightarrow$ BD = 3.6 cm $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Prove that: $\sqrt{\sec^2\theta + \text{cosec}^2\theta}$ = tan θ + cot θ

Sol. LHS = $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$ 1

$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta}$

$= \sqrt{(\tan\theta + \cot\theta)^2}$ $\frac{1}{2}$

= tan θ + cot θ = RHS $\frac{1}{2}$

OR

Prove that: $\frac{\sin\theta}{1 - \cos\theta}$ = (cosec θ + cot θ)

Sol. LHS = $\frac{\sin\theta}{1 - \cos\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$ 1

$= \frac{\sin\theta(1 + \cos\theta)}{1 - \cos^2\theta}$ $\frac{1}{2}$

$= \frac{\sin\theta(1 + \cos\theta)}{\sin^2\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

= cosec θ + cot θ = RHS $\frac{1}{2}$

---

22. Find the values of p for which the quadratic equation $x^2 - 2px + 1 = 0$ has no real roots.

Sol. For no real roots

$D < 0$

$(-2p)^2 - 4 \times 1 \times 1 < 0$ 1

$p^2 - 1 < 0$ $\frac{1}{2}$

$-1 < p < 1$ $\frac{1}{2}$

---

23. Two dice are thrown at the same time. Find the probability of getting different numbers on the two dice.

Sol. Total number of outcomes = 36 $\frac{1}{2}$

Favourable numbers of outcomes = 30 $\frac{1}{2}$

Probability = $\frac{30}{36}$ or $\frac{5}{6}$ 1

(Both no. are dfiferent)

OR

Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is more than 9.

Sol. Favourable outcomes (5, 5), (4, 6), (6, 4), (6, 5), (5, 6), (6, 6)

Total number of outcomes = 36 $\frac{1}{2}$

Number of favourable outcomes = 6 $\frac{1}{2}$

Required probability = $\frac{6}{36}$ or $\frac{1}{6}$ 1

---

24. A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is

(i) red or white

(ii) not a white ball

Sol. Total no. of balls = 20

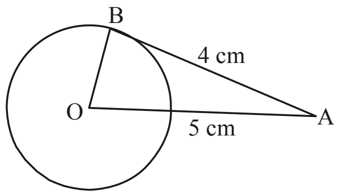
(i) P(ball is red or white) = $\frac{13}{20}$ 1

(ii) P(Not a white ball) = $\frac{12}{20}$ or $\frac{3}{5}$ 1

---

25. The length of a tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. Find the diameter of the circle.

Sol.



OA = 5 cm

AB = 4 cm $\frac{1}{2}$

$OB^2 = OA^2 - AB^2$

$= 25 - 16$

$= 9$ 1

OB = 3 cm

diameter = 6 cm $\frac{1}{2}$

---

26. Find the area of a circle whose circumference is 44 cm.

Sol. $2\pi r = 44$

$r = 7$ cm 1

Area of circle = $\frac{22}{7} \times 7 \times 7 = 154\ \text{cm}^2$ 1

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. In Fig. 3, arrangement of desks in a classroom is shown. Ashima, Bharti and Asha are seated at A, B and C respectively. Answer the following:

(i) Find whether the girls are sitting in a line.

(ii) If A, B and C are collinear, find the ratio in which point B divides the line segment joining A and C.

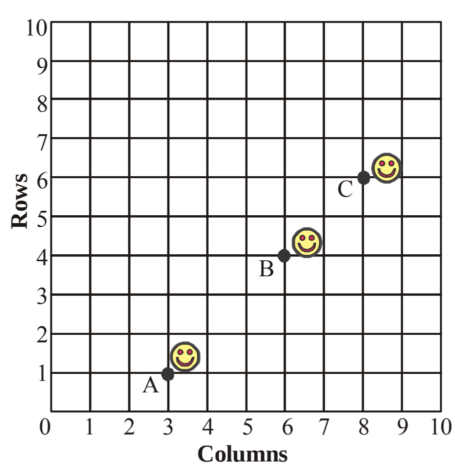


Fig. 3

Sol. Coordinates of A(3, 1)

B(6, 4)

C(8, 6) 1

(i) Area of (ΔABC) $= \frac{1}{2}[3(4-6)+6(6-1)+8(1-4)]$ $\frac{1}{2}$

$= 0$

Yes they are sitting in same line $\frac{1}{2}$

(ii) Let AB:BC = k : 1

$6 = \frac{8k+3}{k+1}$ $\frac{1}{2}$

$k = \frac{3}{2}$ or $\text{Ratio} = 3:2$ $\frac{1}{2}$

---

28. A number consists of two digits whose sum is 10. If 18 is subtracted from the number, its digit are reversed. Find the number.

Sol. Let two digit number = 10x + y $\frac{1}{2}$

$x + y = 10$ ...(i) $\frac{1}{2}$

$10x + y - 18 = 10y + x$

$\Rightarrow$ $x - y = 2$ ...(ii) 1

On solving (i) & (ii) $x = 6$, $y = 4$ $\frac{1}{2}$

$\therefore$ Required number = 64 $\frac{1}{2}$

---

29. If $\sqrt{2}$ is given as an irrational number, then prove that $(7 - 2\sqrt{2})$ is an irrational number.

Sol. Let $7-2\sqrt{2} = m$, where m is a rational number $\frac{1}{2}$

$\sqrt{2} = \frac{7-m}{2}$ 1

Irrational = Rational 1

$\Rightarrow$ LHS $\neq$ RHS

It means out assumption is wrong.

Hence, $7-2\sqrt{2}$ is irrational $\frac{1}{2}$

OR

Find HCF of 44, 96 and 404 by prime factorization method. Hence find their LCM.

Sol. $\left.\begin{array}{l} 44 = 2^2 \times 11 \\ 96 = 2^5 \times 3 \\ 404 = 2^2 \times 101 \end{array}\right]$ $1\frac{1}{2}$

HCF = 2^2 = 4 $\frac{1}{2}$

LCM = $2^5 \times 11 \times 3 \times 101$

= 106656 1

---

30. If 1 and –2 are the zeroes of the polynomial ($x^3 - 4x^2 - 7x + 10$), find its third zero.

Sol. The two factors of polynomials are (x – 1), (x + 2) $\frac{1}{2}$

$(x - 1)(x + 2) = x^2 + x - 2$ $\frac{1}{2}$

$\frac{x^3 - 4x^2 - 7x + 10}{x^2 + x - 2} = (x - 5)$ $1\frac{1}{2}$

Third zero = 5 $\frac{1}{2}$

---

31. Draw a circle of radius 3 cm. From a point 7 cm away from its centre, construct a pair of tangents to the circle.

Sol. Drawing a circle of radius 3 cm, marking Centre 0 and taking a point P such that OP = 7 cm 1

Constructing two tangents 2

OR

Draw a line segment of 8 cm and divide it in the ratio 3 : 4.

Sol. Drawing a line segment of 8 cm 1

Dividing it in the ratio 3 : 4 2

---

32. **Prove that** $\dfrac{\cos\theta}{(1-\tan\theta)}+\dfrac{\sin\theta}{(1-\cot\theta)}$ **= (cos θ + sin θ)**

Sol. LHS = $\dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin\theta}{1-\cot\theta}$

$= \dfrac{\cos^2\theta}{\cos\theta-\sin\theta}+\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}$ 1

$= \dfrac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$ 1

$= \cos\theta + \sin\theta$ = RHS 1

OR

Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$**.**

Sol. $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \text{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$ $\frac{1}{2}+\frac{1}{2}$

$= \sin^2\theta + 1 + \cot^2\theta + 2 + \cos^2\theta + 1 + \tan^2\theta + 2$ $\frac{1}{2}+\frac{1}{2}$

$= 7 + \tan^2\theta + \cot^2\theta$ 1

---

33. **In Fig. 3, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle and AB is tangent at R. Prove that:**

XA + AR = XB + BR

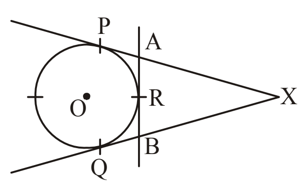


Fig. 3

Sol. XP = XQ (tangents from external points) 1

XA + AP = XB + BQ 1

XA + AR = XB + BR (AP = AR, BQ = BR) 1

---

34. The radii of two circles are 8 cm and 6 cm. Find the radius of the circle having its area equal to the sum of the areas of the two circles.

Sol. Let r be the radius of required circle

Here, $\pi(8)^2 + \pi(6)^2 = \pi r^2$ 1

$100 = r^2$ 1

$r = 10$ cm 1

SECTION D

Question Nos. 35 to 40 carry 4 marks each.

35. In a right triangle, prove that the square of the hypotenuse is equal to sum of squares of the other two sides.

Sol. For correct given, to prove, construction and figure $4\times\frac{1}{2}=2$

For correct proof 2

OR

Prove that the tangents drawn from an external point to a circle are equal in length.

Sol. For correct given, to prove, construction and figure $4\times\frac{1}{2}=2$

For correct proof 2

36. A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to edge of the cube. Determine the volume of the remaining block.

Sol. Let r be the radius of hemisphere $\therefore\ r = \frac{21}{2}$ cm $\frac{1}{2}$

Volume of remaining block $= a^3 - \frac{2}{3}\pi r^3$

$= (21)^3 - \frac{2}{3}\pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$ 2

$= 9261\left[1 - \frac{\pi}{12}\right] \text{cm}^3$ 1

$= 6853$ cm^3 (Approx.) $\frac{1}{2}$

OR

A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere of same radius. Find the radius of the hemisphere and total height of the toy, if the height of the cone is 3 times the radius.

Sol. Here, r = 6 cm

$\pi(6)^2 \times 15 = 12\left[\frac{1}{3}\pi r^2 \times 3r + \frac{2}{3}\pi r^3\right]$ 2

$36 \times 15 = \frac{12}{3}[3r^3 + 2r^3]$ $\frac{1}{2}$

$9 \times 15 = 5r^3$

r = 3 cm $\frac{1}{2}$

Total height = 12 cm 1

---

37. The sum of first 6 terms of an A.P. is 42. The ratio of its 10th term to 30th term is 1:3. Find the first and the 13th term of the A.P.

Sol. Here, $\frac{6}{2}(2a + 5d) = 42$

$\Rightarrow$ 2a + 5d = 14 ...(i) 1

Also,

$\frac{a + 9d}{a + 29d} = \frac{1}{3}$ ...(ii) 1

$\Rightarrow$ a = d $\frac{1}{2}$

Solving (i) and (ii), 7a = 14 $\frac{1}{2}$

$\Rightarrow$ a = 2

d = 2 $\frac{1}{2}$

$a_{13} = a + 12d = 26$ $\frac{1}{2}$

OR

Find the sum of all odd numbers between 100 and 300.

Sol. Odd number between 100 to 300 are 1

101, 103 ... 299

$299 = 101 + (n - 1)2$

$\Rightarrow \quad n = 100$ 1

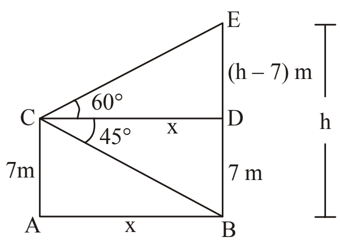
$S_n = \frac{100}{2}(101+299)$ 1

$= 20{,}000$ 1

---

38. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60°, and the angle of depression of its foot is 45°. Find the height of the tower. Given that $\sqrt{3}$ = 1.732.

Sol.



Correct figure 1

$\tan 45° = \frac{7}{x}$

$\Rightarrow x = 7$ m ...(i) 1

$\tan 60° = \frac{h-7}{x}$

$x\sqrt{3} = h - 7$ ....(ii) 1

Solving (i) and (ii) $h = 7(\sqrt{3}+1)$

$= 7 \times 2.732$

$= 19.124$ m 1

---

39. Find the mean of the following distribution:

Classes:	**100 – 150**	**150 – 200**	**200 – 250**	**250 – 300**	**300 – 350**
Frequency:	**4**	**5**	**12**	**2**	**2**

Sol.

CI	f_i	x_i	d_i	u_i	f_iu_i
100-150	4	125	–100	–2	–8
150-200	5	175	–50	–1	–5
200-250	12	225	0	0	0
250-300	2	275	50	1	2
300-350	2	325	100	2	4
Total	25				–7

Correct Table 2

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 225 + \frac{-7}{25} \times 50$$ 1

$$= 211$$ 1

---

40. The sum of the reciprocals of the ages of a child 3 years ago and 5 years hence from now is $\frac{1}{3}$. Find his present age.

Sol. Let the present age = x years

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$ 2

$\Rightarrow$ $x^2 - 4x - 21 = 0$ 1

$(x - 7)(x + 3) = 0$

Hence, present age = 7 years 1

QUESTION PAPER CODE 430/3/3
EXPECTED ANSWER/VALUE POINTS
SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

Select the correct choice.

1. The simplest form of $\frac{1095}{1168}$ is

(a) $\frac{17}{26}$ (b) $\frac{25}{26}$ (c) $\frac{13}{16}$ (d) $\frac{15}{16}$

Sol. (d) $\frac{15}{16}$ 1

---

2. One card is drawn at random from a well – shuffled deck of 52 cards. What is the probability of getting a Jack?

(a) $\frac{3}{26}$ (b) $\frac{1}{52}$ (c) $\frac{1}{13}$ (d) $\frac{3}{52}$

Sol. (c) $\frac{1}{13}$ 1

---

3. If one zero of the quadratic polynomial, $(k - 1) x^2 + kx + 1$ is -4 then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{5}{4}$ (c) $-\frac{4}{3}$ (d) $\frac{4}{3}$

Sol. (b) $\frac{5}{4}$ 1

---

4. If P(–1, 1) is the midpoint of the line segment joining A(–3, b) and B(1, b + 4), then b is equal to

(a) 1 (b) –1 (c) 2 (d) 0

Sol. (b) –1 1

---

5. Which of the following rational numbers is expressible as a terminating decimal?

(a) $\frac{124}{165}$ (b) $\frac{131}{30}$ (c) $\frac{2027}{625}$ (d) $\frac{1625}{462}$

Sol. (c) $\frac{2027}{625}$ 1

---

6. **Consider the following distribution:**

Classes:	**0 – 5**	**5 – 10**	**10 – 15**	**15 – 20**	**20 – 25**
Frequency:	**10**	**15**	**12**	**20**	**9**

The sum of lower limits of the median class and the modal class is

(a) 15 **(b) 25** **(c) 30** **(d) 35**

Sol. (b) 25 1

---

7. **What is the largest number that divides 245 and 1029, leaving remainder 5 in each?**

(a) 15 **(b) 16** **(c) 9** **(d) 5**

Sol. (b) 16 1

---

8. **If PA and PB are tangents to a circle with centre O such that $\angle APB = 70°$, then $\angle AOB$ is**

(a) 140° **(b) 110°** **(c) 35°** **(d) 70°**

Sol. (b) 110° 1

---

9. **If α and β are the zeroes of the polynomial $3x^2 + 4x - 3$, then value of $\alpha\beta$ is**

(a) 1 **(b) $\frac{4}{3}$** **(c) $-\frac{4}{3}$** **(d) –1**

Sol. (d) –1 1

---

10. **Distance of the point $(a \cos \theta, a \sin \theta)$ from origin is:**

(a) a **(b) a^2** **(c) ±a** **(d) 1**

Sol. (a) a 1

In Question numbers 11 to 15, fill in the blanks:

11. **The value of $(\tan 27° - \cot 63°)$ is ________.**

Sol. 0 1

---

12. **If ratio of the corresponding sides of two similar triangles is 2:3, then ratio of their perimeters is __________.**

Sol. 2 : 3 1

---

13. **The value of k for which system of equations $x + 2y = 3$ and $5x + ky = 7$ has no solution is ________.**

Sol. k = 10 1

---

14. **Distance between (a, –b) and (a, b) is ________.**

Sol. 2b units 1

---

15. **The value of $(\sec^2 20° - \cot^2 70°)$ is ________.**

Sol. 1 — 1

Answer the following questions, Question numbers 16 to 20.

16. **The perimeter of a sector of a circle of radius 14 cm is 68 cm. Find the area of the sector.**

Sol. $l = 68 - 28 = 40$ cm — $\frac{1}{2}$

$A = 280 \text{ cm}^2$ — $\frac{1}{2}$

OR

The circumference of a circle is 39.6 cm. Find its area.

Sol. $r = \dfrac{39.6}{2\pi}$ — $\frac{1}{2}$

$A = \dfrac{392.04}{\pi} \text{ or } 124.74 \text{ cm}^2$ — $\frac{1}{2}$

---

17. **If $\sec\theta = \dfrac{25}{7}$, then find the value of $\cot\theta$.**

Sol. $\tan\theta = \dfrac{24}{7} \Rightarrow \cot\theta = \dfrac{7}{24}$ — $\frac{1}{2} + \frac{1}{2}$

OR

If $3\tan\theta = 4$, then find the value of $\left(\dfrac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}\right)$

Sol. Given expression $= \dfrac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = 3$ — $\frac{1}{2} + \frac{1}{2}$

---

18. **If $3y - 1$, $3y + 5$ and $5y + 1$ are three consecutive terms of an A.P., then find the value of y.**

Sol. $2(3y + 5) = 3y - 1 + 5y + 1$ — $\frac{1}{2}$

$y = 5$ — $\frac{1}{2}$

---

19. **In Fig. 1, DE || BC, AD = 3 cm and BD = 2 cm;**

Find $\dfrac{\text{ar}\,(\Delta ADE)}{\text{ar}\,(\Delta ABC)}$

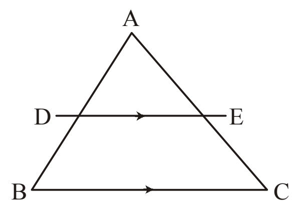


Fig. 1

Sol. AB = 3 + 2 = 5 cm $\frac{1}{2}$

$$\frac{\text{ar (ADE)}}{\text{ar (ABC)}} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$ $\frac{1}{2}$

---

20. **A bag contains 4 red, 5 white and 6 green balls. A ball is drawn at random from the bag. Find the probability of getting not a red ball.**

Sol. Total No. of balls = 15 $\frac{1}{2}$

P(Not a red ball) = $\frac{11}{15}$ $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. **Prove that:** $\sqrt{\sec^2\theta + \text{cosec}^2\theta}$ **= tan θ + cot θ**

Sol. LHS = $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$ 1

$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta}$

$= \sqrt{(\tan\theta + \cot\theta)^2}$ $\frac{1}{2}$

$= \tan\theta + \cot\theta$ = RHS $\frac{1}{2}$

OR

Prove that: $\dfrac{\sin\theta}{1-\cos\theta} = (\text{cosec } \theta + \cot \theta)$

Sol. LHS $= \dfrac{\sin\theta}{1-\cos\theta} \times \dfrac{1+\cos\theta}{1+\cos\theta}$ 1

$= \dfrac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta}$ $\frac{1}{2}$

$= \dfrac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta} = \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \text{cosec } \theta + \cot \theta = \text{RHS}$ $\frac{1}{2}$

---

22. A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is

(i) red or white

(ii) not a white ball

Sol. Total no. of balls = 20

(i) P(ball is red or white) $= \dfrac{13}{20}$ 1

(ii) P(Not a white ball) $= \dfrac{12}{20}$ or $\dfrac{3}{5}$ 1

---

23. Find the values of p for which the quadratic equation $x^2 - 2px + 1 = 0$ has no real roots.

Sol. For no real roots

$D < 0$

$(-2p)^2 - 4 \times 1 \times 1 < 0$ 1

$p^2 - 1 < 0$ $\frac{1}{2}$

$-1 < p < 1$ $\frac{1}{2}$

---

24. Two dice are thrown at the same time. Find the probability of getting different numbers on the two dice.

Sol. Total number of outcomes = 36 $\frac{1}{2}$

Favourable numbers of outcomes = 30 $\frac{1}{2}$

Probability $= \dfrac{30}{36}$ or $\dfrac{5}{6}$ 1

(Both numbers are different)

OR

Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is more than 9.

Sol. Favourable outcomes (5, 5), (4, 6), (6, 4), (6, 5), (5, 6), (6, 6)

Total number of outcomes = 36 $\frac{1}{2}$

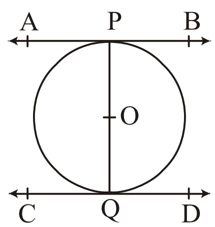
Number of favourable outcomes = 6 $\frac{1}{2}$

Required probability = $\frac{6}{36}$ or $\frac{1}{6}$ 1

---

25. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol.



Correct figure $\frac{1}{2}$

$$\left.\begin{aligned}\angle OPA = 90^\circ\\ \angle OQD = 90^\circ\end{aligned}\right\}\left\{\text{radius is perpendicular to tan gent}\right\}$$ 1

But they are forming alternate interior angle

$\Rightarrow$ AB || CD $\frac{1}{2}$

---

26. In Fig. 2, OACB is a quadrant of a circle with Centre O and radius 7 cm. If OD = 4 cm, find the area of the shaded region.

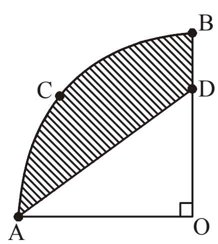


Fig. 2

Sol. Area of shaded region = $\frac{1}{4}\pi(7)^2 - \frac{1}{2}\times 7\times 4$ 1

$= \left(\frac{49}{4}\pi - 14\right)\text{cm}^2$ $\frac{1}{2}$

$= 24.5\ \text{cm}^2$ $\frac{1}{2}$

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. A number consists of two digits whose sum is 10. If 18 is subtracted from the number, its digit are reversed. Find the number.

Sol. Let two digit number = $10x + y$ $\frac{1}{2}$

$x + y = 10$ ...(i) $\frac{1}{2}$

$10x + y - 18 = 10y + x$

$\Rightarrow$ $x - y = 2$ ...(ii) 1

On solving (i) & (ii) $x = 6$, $y = 4$ $\frac{1}{2}$

$\therefore$ Required number = 64 $\frac{1}{2}$

---

28. If 1 and –2 are the zeroes of the polynomial ($x^3 - 4x^2 - 7x + 10$), find its third zero.

Sol. The two factors of polynomials are $(x - 1)$, $(x + 2)$ $\frac{1}{2}$

$(x - 1)(x + 2) = x^2 + x - 2$ $\frac{1}{2}$

$$\frac{x^3 - 4x^2 - 7x + 10}{x^2 + x - 2} = (x - 5)$$ $1\frac{1}{2}$

Third zero = 5 $\frac{1}{2}$

---

29. Draw a circle of radius 3 cm. From a point 7 cm away from its centre, construct a pair of tangents to the circle.

Sol. Drawing a circle of radius 3 cm, marking Centre 0 and taking a point P such that OP = 7 cm 1

Constructing two tangents 2

OR

Draw a line segment of 8 cm and divide it in the ratio 3 : 4.

Drawing a line segment of 8 cm 1

Dividing it in the ratio 3 : 4 2

---

30. **In Fig. 3, arrangement of desks in a classroom is shown. Ashima, Bharti and Asha are seated at A, B and C respectively. Answer the following:**

(i) Find whether the girls are sitting in a line.

(ii) If A, B and C are collinear, find the ratio in which point B divides the line segment joining A and C.

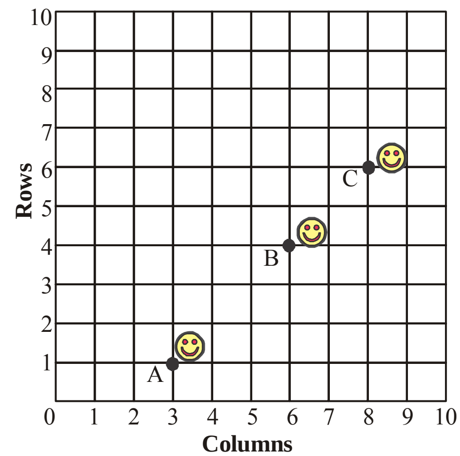


Fig. 3

Sol. Coordinates of A(3, 1)

B(6, 4)

C(8, 6) 1

(i) Area of $(\Delta ABC) = \frac{1}{2}[3(4-6)+6(6-1)+8(1-4)]$ $\frac{1}{2}$

$= 0$

Yes they are sitting in same line $\frac{1}{2}$

(ii) Let AB : BC = k : 1

$6 = \frac{8k+3}{k+1}$ $\frac{1}{2}$

$k = \frac{3}{2}$ or Ratio $= 3:2$ $\frac{1}{2}$

---

31. **Prove that** $\frac{\cos\theta}{(1-\tan\theta)} + \frac{\sin\theta}{(1-\cot\theta)} = (\cos\theta + \sin\theta)$

Sol. LHS $= \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$

$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$$ 1

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$$ 1

$= \cos\theta + \sin\theta = \text{RHS}$ 1

OR

Prove that $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

Sol. $(\sin\theta + \operatorname{cosec}\theta) + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \operatorname{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$ $\frac{1}{2}+\frac{1}{2}$

$= \sin^2\theta + 1 + \cot^2\theta + 2 + \cos^2\theta + 1 + \tan^2\theta + 2$ $\frac{1}{2}+\frac{1}{2}$

$= 7 + \tan^2\theta + \cot^2\theta$ 1

---

32. If $\sqrt{2}$ is given as an irrational number, then prove that $(7 - 2\sqrt{2})$ is an irrational number.

Sol. Let $7 - 2\sqrt{2} = \text{m},$ where m is a rational number $\frac{1}{2}$

$$\sqrt{2} = \frac{7 - \text{m}}{2}$$ 1

Irrational = Rational 1

$\Rightarrow$ LHS $\neq$ RHS

It means out assumption is wrong.

Hence, $7 - 2\sqrt{2}$ is irrational $\frac{1}{2}$

OR

Find HCF of 44, 96 and 404 by prime factorization method. Hence find their LCM.

Sol.

$$\left.\begin{array}{l} 44 = 2^2 \times 11 \\ 96 = 2^5 \times 3 \\ 404 = 2^2 \times 101 \end{array}\right]$$ $1\frac{1}{2}$

HCF = 2^2 = 4 $\frac{1}{2}$

LCM = $2^5 \times 11 \times 3 \times 101$

= 106656 1

---

33. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m × 14 m. Find the height of the platform.

Sol. Let height of platform be h m

$$\therefore \quad \pi\left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times h$$ 2

$$\Rightarrow \quad h = \frac{35}{44}\pi$$ 1

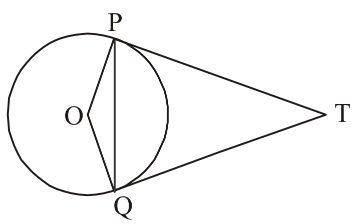
OR

h = 2.5 m

---

34. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ.

Sol.



Correct figure $\frac{1}{2}$

$\angle OPQ + \angle QPT = 90°$ ...(i) 1

$\angle PTQ = 180° - 2\angle QPT$ ...(ii) 1

By (i) & (ii)

$\angle PTQ = 180° - 2(90° - \angle OPQ)$

$\angle PTQ = 2\angle OPQ$ $\frac{1}{2}$

SECTION D

Question Nos. 35 to 40 carry 4 marks each.

35. In a right triangle, prove that the square of the hypotenuse is equal to sum of squares of the other two sides.

Sol. For correct given, to prove, construction and figure $4\times\frac{1}{2}=2$

For correct proof 2

OR

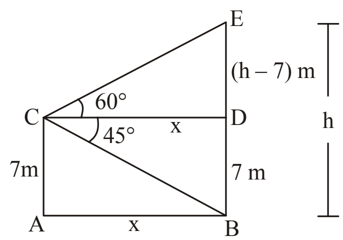
Prove that the tangents drawn from an external point to a circle are equal in length.

Sol. For correct given, to prove, construction and figure $4\times\frac{1}{2}=2$

For correct proof 2

36. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60°, and the angle of depression of its foot is 45°. Find the height of the tower. Given that $\sqrt{3}$ = 1.732.

Sol.



Correct figure 1

$$\tan 45° = \frac{7}{x}$$

$\Rightarrow x = 7$ m ...(i) 1

$$\tan 60° = \frac{h-7}{x}$$

$x\sqrt{3} = h - 7$ ....(ii) 1

Solving (i) and (ii), $h = 7(\sqrt{3}+1)$

$= 7 \times 2.732$

$= 19.124$ m 1

---

37. The sum of first 6 terms of an A.P. is 42. The ratio of its 10th term to 30th term is 1:3. Find the first and the 13th term of the A.P.

Sol. Here, $\frac{6}{2}(2a+5d) = 42$

$\Rightarrow 2a + 5d = 14$ ...(i) 1

Also,

$$\frac{a+9d}{a+29d} = \frac{1}{3}$$ ...(ii) 1

$\Rightarrow a = d$ $\frac{1}{2}$

Solving (i) and (ii), $7a = 14$ $\frac{1}{2}$

$\Rightarrow a = 2$

$d = 2$ $\frac{1}{2}$

$a_{13} = a + 12d = 26$ $\frac{1}{2}$

OR

Find the sum of all odd numbers between 100 and 300.

Sol. Odd number between 100 to 300 are 1

$101, 103 \ldots 299$

$299 = 101 + (n - 1)2$

$\Rightarrow n = 100$ 1

$S_n = \frac{100}{2}(101 + 299)$ 1

$= 20{,}000$ 1

---

38. A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to edge of the cube. Determine the volume of the remaining block.

Sol. Let r be the radius of hemisphere $\therefore r = \frac{21}{2}$ cm $\frac{1}{2}$

Volume of remaining block = $a^3 - \frac{2}{3}\pi r^3$

$= (21)^3 - \frac{2}{3}\pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$ 2

$= 9261\left[1 - \frac{\pi}{12}\right] \text{cm}^3$ 1

$= 6853 \text{ cm}^3$ (Approx.) $\frac{1}{2}$

OR

A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere of same radius. Find the radius of the hemisphere and total height of the toy, if the height of the cone is 3 times the radius.

Sol. Here, $r = 6$ cm

$\pi(6)^2 \times 15 = 12\left[\frac{1}{3}\pi r^2 \times 3r + \frac{2}{3}\pi r^3\right]$ 2

$36 \times 15 = \frac{12}{3}[3r^3 + 2r^3]$ $\frac{1}{2}$

$9 \times 15 = 5r^3$

$r = 3$ cm $\frac{1}{2}$

Total height = 12 cm 1

---

39. **The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.**

Sol. Let the numbers are x, y (x > y)

$x^2 - y^2 = 180$ 1

$y^2 = 8x$ 1

On solving $x^2 - 8x - 180 = 0$ $\frac{1}{2}$

$(x - 1)(x + 10) = 0$

$x = 18, -10$ (rejected) $\frac{1}{2}$

Numbers are 18, 12 or 18, –12 $\frac{1}{2}+\frac{1}{2}$

---

40. **Find the mean of the following frequency distribution :**

Classes	**5 – 15**	**15 – 25**	**25 – 35**	**35 – 45**	**45 – 55**	**55 – 65**
Frequency	**6**	**11**	**21**	**23**	**14**	**5**

Sol.

CI	**f_i**	**x_i**	**d_i**	**u_i**	**f_iu_i**
5-15	6	10	–20	–2	–12
15-25	11	20	–10	–1	–11
25-35	21	30	0	0	0
35-45	23	40	10	1	23
45-55	14	50	20	2	28
55-65	5	60	30	3	15
Total	80				43

Correct Table 2

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 30 + \frac{43}{80} \times 10$$ 1

$$= 35.375$$ 1