

Secondary School Examination - 2020
Marking Scheme- MATHEMATICS BASIC
Subject Code : 241 Paper Code: 430/2/1,2,3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-X, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(✓) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks **0 - 80** has to be used. Please do not hesitate to award full marks if the answer deserves it.

10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
 - Leaving answer or part thereof unassessed in an answer book.
 - Giving more marks for an answer than assigned to it.
 - Wrong totaling of marks awarded on a reply.
 - Wrong transfer of marks from the inside pages of the answer book to the title page.
 - Wrong question wise totaling on the title page.
 - Wrong totaling of marks of the two columns on the title page.
 - Wrong grand total.
 - Marks in words and figures not tallying.
 - Wrong transfer of marks from the answer book to online award list.
 - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
 - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 430/2/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option.

1. HCF of two numbers is 27 and their LCM is 162. If one of the number is 54, then the other number is

(a) 36 (b) 35 (c) 9 (d) 81

Sol. (d) 81 1

2. The cumulative frequency table is useful in determining

(a) Mean (b) Median (c) Mode (d) All of these

Sol. (b) Median 1

3. In Fig. 1, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with PQ. $\angle POQ$ is

(a) 130° (b) 90° (c) 100° (d) 75°

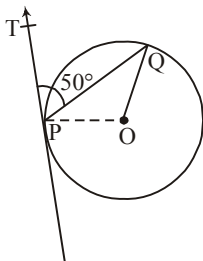


Fig. 1

Sol. (c) 100° 1

4. $2\sqrt{3}$ is

(a) an integer (b) a rational number
 (c) an irrational number (d) a whole number

Sol. (c) an irrational no. 1

5. Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Sol. (d) $\frac{3}{4}$ 1

6. If one zero of the polynomial $(3x^2 + 8x + k)$ is the reciprocal of the other, then value of k is

- (a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

Sol. (a) 3 1

7. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?

- (a) 2 (b) 4 (c) 5 (d) 1

Sol. (c) 5 1

8. The maximum number of zeroes a cubic polynomial can have, is

- (a) 1 (b) 4 (c) 2 (d) 3

Sol. (d) 3 1

9. The distance of the point $(-12, 5)$ from the origin is

- (a) 12 (b) 5 (c) 13 (d) 169

Sol. (c) 13 1

10. If the centre of a circle is $(3, 5)$ and end points of a diameter are $(4, 7)$ and $(2, y)$, then the value of y is

- (a) 3 (b) -3 (c) 7 (d) 4

Sol. (a) 3 1

Question numbers 11 to 15, fill in the blanks:

11. The area of triangle formed with the origin and the points $(4, 0)$ and $(0, 6)$ is _____.

Sol. 12 sq units 1

OR

The co-ordinate of the point dividing the line segment joining the points $A(1, 3)$ and $B(4, 6)$ in the ratio $2 : 1$ is _____.

Sol. $(3, 5)$ 1

12. Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are _____.

Sol. 3 and -2 1

13. If $\sin \theta = \frac{5}{13}$, then the value of $\tan \theta$ is _____.

Sol. $\tan \theta = \frac{5}{12}$ 1

14. The value of $(\tan^2 60^\circ + \sin^2 45^\circ)$ is _____.

Sol. $\frac{7}{2}$ or 3.5

1

15. The corresponding sides of two similar triangles are in the ratio 3 : 4, then the ratios of the area of triangles is _____.

Sol. 9 : 16

1

Question numbers 16 to 20, answer the following :

16. Find the value of $(\cos 48^\circ - \sin 42^\circ)$.

Sol. $\cos 48^\circ - \cos (90 - 42^\circ)$

 $\frac{1}{2}$

$$\cos 48^\circ - \cos 48^\circ = 0$$

 $\frac{1}{2}$

OR

Evaluate: $(\tan 23^\circ) \times (\tan 67^\circ)$

Sol. $\tan (90 - 67^\circ) \times \tan 67^\circ$

 $\frac{1}{2}$

$$\cot 67^\circ \times \tan 67^\circ$$

 $\frac{1}{2}$

$$= 1$$

17. In figure-2 \widehat{PQ} and \widehat{AB} are two arcs of concentric circles of radii 7 cm and 3.5 cm resp., with centre O. If $\angle POQ = 30^\circ$, then find the area of shaded region.

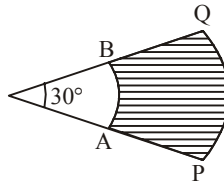


Fig.-2

Sol. Area of shaded region = $\frac{22}{7} \times \frac{30^\circ}{360^\circ} (7^2 - (3.5)^2)$

 $\frac{1}{2}$

$$= 9.625 \text{ cm}^2$$

 $\frac{1}{2}$

18. A card is drawn at random from a well shuffled deck of 52 playing cards. What is the probability of getting a black king?

Sol. $P(\text{Black king}) = \frac{2}{52}$ or $\frac{1}{26}$

1

19. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Sol. Distance = $\sqrt{(25)^2 - (24)^2} = 7 \text{ m}$

 $\frac{1}{2} + \frac{1}{2}$

20. If $3k - 2$, $4k - 6$ and $k + 2$ are three consecutive terms of A.P., then find the value of k .

Sol. $(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$ $\frac{1}{2}$

$\Rightarrow k = 3$ $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol. Total = $10 + 25 = 35$, $P(\text{getting prize}) = \frac{10}{35}$ or $\frac{2}{7}$ 1+1

22. In a family of three children, find the probability of having at least two boys.

Sol. Total outcomes = 8 {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} 1

$P(\text{atleast 2 boys}) = \frac{4}{8}$ or $\frac{1}{2}$ 1

OR

Two dice are tossed simultaneously. Find the probability of getting

(i) an even number on both dice.

(ii) the sum of two numbers more than 9.

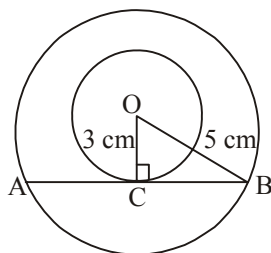
Total outcomes = 36 1

$P(\text{even no. on both side}) = \frac{9}{36}$ or $\frac{1}{4}$ $\frac{1}{2}$

$P(\text{sum} > 9) = \frac{6}{36}$ or $\frac{1}{6}$ $\frac{1}{2}$

23. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle which touches the smaller circle.

Sol.



In $\triangle OCB$

Fig. $\frac{1}{2}$

$BC = \sqrt{5^2 - 3^2} = 4$ cm 1

$AB = 2 \times BC = 8$ cm $\frac{1}{2}$

24. Prove that: $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Sol. L.H.S = $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$ 1

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$$
 $\frac{1}{2}$

$$= 2\sec^2\theta$$
 $\frac{1}{2}$

OR

Prove that: $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Sol. L.H.S = $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$ 1

$$= \cos^2\theta - \sin^2\theta$$
 1

25. The wheel of a motorcycle is of radius 35 cm. How many revolutions are required to travel a distance of 11 m?

Sol. Distance in 1 revolution = $2 \times \frac{22}{7} \times 35 = 220$ cm 1

No. of revolution = $\frac{1100}{220} = 5$ 1

26. Divide $(2x^2 - x + 3)$ by $(2 - x)$ and write the quotient and the remainder.

Sol.
$$\begin{array}{r} -2x-3 \\ -x+2 \overline{) 2x^2 - x + 3} \\ \underline{2x^2 - 4x} \\ - + \\ 3x + 3 \\ \underline{3x - 6} \\ - 9 \end{array}$$
 1

Quotient = $-2x - 3$ 1

R = 9 1

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. If α and β are the zeroes of the polynomial $f(x) = 5x^2 - 7x + 1$, then find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

Sol. $\alpha + \beta = \frac{7}{5}$ and $\alpha\beta = \frac{1}{5}$ $\frac{1}{2} + \frac{1}{2}$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \quad 1$$

$$= \frac{\left(\frac{7}{5}\right)^2 - 2 \times \frac{1}{5}}{\frac{1}{5}} \quad \frac{1}{2}$$

$$= \frac{39}{5} \text{ or } 7.8 \quad \frac{1}{2}$$

28. Draw a line segment of length 7 cm and divide it in the ratio 2 : 3.

Sol. Correct construction 3

OR

Draw a circle of radius 4 cm and construct the pair of tangents to the circle from an external point, which is at a distance of 7 cm from its centre.

Sol. Correct construction 3

29. The minute hand of a clock is 21 cm long. Calculate the area swept by it and the distance travelled by its tip in 20 minutes.

Sol. Angle in 20 min = 120° $\frac{1}{2}$

$$\text{Area} = \frac{22}{7} \times \frac{120^\circ}{360^\circ} \times (21)^2 = 462 \text{ cm}^2 \quad 1 + \frac{1}{2}$$

$$\text{Distance} = \frac{120^\circ}{360^\circ} \times 2\pi r = 44 \text{ cm} \quad 1$$

30. If $x = 3 \sin \theta + 4 \cos \theta$ and $y = 3 \cos \theta - 4 \sin \theta$ then prove that $x^2 + y^2 = 25$.

Sol. $x^2 = 9 \sin^2 \theta + 16 \cos^2 \theta + 24 \sin \theta \cos \theta$ 1
 $y^2 = 9 \cos^2 \theta + 16 \sin^2 \theta - 24 \sin \theta \cos \theta$ 1
 $x^2 + y^2 = 25$ 1

OR

If $\sin \theta + \sin^2 \theta = 1$; then prove that $\cos^2 \theta + \cos^4 \theta = 1$.

Sol. $\sin \theta = 1 - \sin^2 \theta = \cos^2 \theta$ 1
L.H.S = $\cos^2 \theta + (\cos^2 \theta)^2 = \cos^2 \theta + \sin^2 \theta$ 1+1
= 1 = R.H.S

31. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ be a rational number

$$\sqrt{3} = \frac{p}{q} \quad p, q \text{ are coprime, } q \neq 0 \quad \frac{1}{2}$$

$$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p \quad \text{Let } p = 3m \quad 1$$

$$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q \quad \frac{1}{2}$$

\therefore 3 is common factor of p and q

Contraction to our assumption 1

Hence $\sqrt{3}$ is irrational No.

OR

Using Euclid's algorithm, find the HCF of 272 and 1032.

Sol. $1032 = 272 \times 3 + 216$

$$272 = 216 \times 1 + 56 \quad \frac{1}{2} + \frac{1}{2}$$

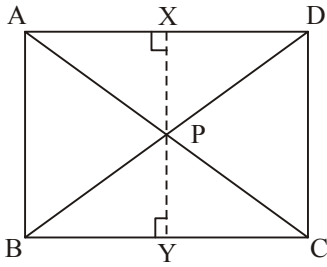
$$216 = 56 \times 3 + 48$$

$$56 = 48 \times 1 + 8 \quad \frac{1}{2} + \frac{1}{2}$$

$$48 = 8 \times 6 + 0 \quad \text{HCF}(1032, 272) = 8 \quad \frac{1}{2} + \frac{1}{2}$$

32. In a rectangle ABCD, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Sol.



Correct figure & Construction

$$\text{In rt } \triangle APX \quad AP^2 = AX^2 + PX^2$$

$$\text{In rt } \triangle PCY \quad PC^2 = PY^2 + YC^2$$

$$\text{In rt } \triangle PBY \quad PB^2 = PY^2 + BY^2$$

$$\text{In rt } \triangle PDX \quad PD^2 = DX^2 + PX^2$$

$$PA^2 + PC^2 = AX^2 + PX^2 + PY^2 + YC^2$$

$$= BY^2 + PY^2 + PX^2 + XD^2$$

$$= PB^2 + PD^2$$

$$\frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$1$$

33. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 3. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

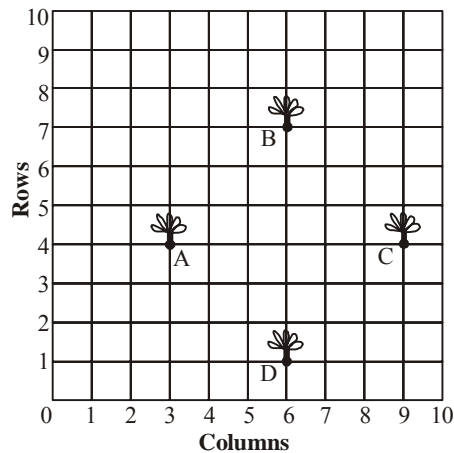


Fig. 3

Sol. $A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1)$

$$AB = 3\sqrt{2}, \quad BC = 3\sqrt{2}, \quad CD = 3\sqrt{2}, \quad DA = 3\sqrt{2}$$

$$AC = 6 \text{ unit} \quad BD = 6 \text{ unit}$$

$$AB = BC = CD = DA \text{ and } AC = BD$$

ABCD is a square

\therefore Champa is correct

$$1$$

$$1$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

34. Solve graphically:

$$2x - 3y + 13 = 0; 3x - 2y + 12 = 0$$

Sol. Correct graph of $2x - 3y + 13 = 0$, $3x - 2y + 12 = 0$

1+1

$$\text{Solution } x = -2, y = 3$$

1

SECTION D

Question numbers 35 to 40 carry 4 marks each.

35. The product of two consecutive positive integers is 306. Find the integers.

Sol. Let two consecutive integers $x, x + 1$

$$x(x + 1) = 306 \Rightarrow x^2 + x - 306 = 0$$

1

$$\Rightarrow (x + 18)(x - 17) = 0$$

1

$$\Rightarrow x = -18, (\text{Rejected}), 17$$

1

\therefore Two consecutive integers 17, 18

1

36. The 17th term of an A.P. is 5 more than twice its 8th term. If 11th term of A.P. is 43; then find its n th term.

Sol. $a_{17} = 2a_8 + 5 \Rightarrow a + 16d = 2(a + 7d) + 5$

1

$$\Rightarrow 2d - a = 15 \quad \dots(1)$$

$$a_{11} = 43 \Rightarrow a + 10d = 43 \quad \dots(2)$$

1

Solving (1) & (2) $a = 3 \quad d = 4$

1

$$a_n = 4n - 1$$

1

OR

How many terms of A.P. 3, 5, 7, 9, ... must be taken to get the sum 120?

Sol. $a = 3, d = 3, S_n = 120$

1

$$\frac{n}{2}[2 \times 3 + (n-1)2] = 120 \Rightarrow n^2 + 2n - 120 = 0$$

1

$$(n + 12)(n - 10) = 0$$

1

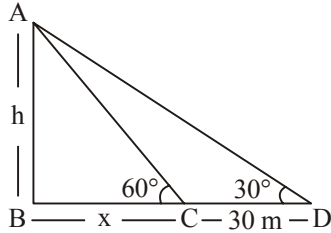
$$n = -12, n = 10$$

1

Reject $n = -12, n = 10$

37. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on opposite bank is 60° . When he moves 30 m away from the bank, he finds the angle of elevation to be 30° . Find the height of the tree and width of the river. [Take $\sqrt{3} = 1.732$]

Sol.



Correct figure

1

In right $\triangle ABC$

$$\tan 60^\circ = \frac{h}{x}$$

 $\frac{1}{2}$

$$\sqrt{3}x = h \quad \dots(1)$$

 $\frac{1}{2}$

$$\text{In rt } \triangle ABD \tan 30^\circ = \frac{h}{30+x} \Rightarrow \frac{30+x}{\sqrt{3}} = h \quad \dots(2) \quad \frac{1}{2} + \frac{1}{2}$$

$$\text{Solving (1) \& (2) } x = 15\text{m, } h = 15\sqrt{3} \text{ m} = 25.98 \text{ m} \quad \frac{1}{2} + \frac{1}{2}$$

38. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Sol. Correct Fig., given, to prove, construction

 $4 \times \frac{1}{2} = 2$

Correct proof given, to prove, construction,

2

OR

Prove that the length of tangents drawn from an external point to a circle are equal.

Correct Fig., given, to prove, construction

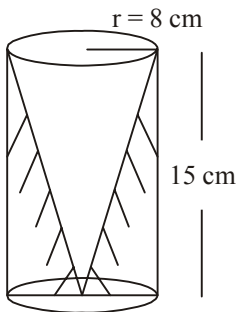
 $4 \times \frac{1}{2} = 2$

Correct proof given, to prove, construction,

2

39. From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of remaining solid. (Give your answer in terms of π).

Sol.

Correct figure $\frac{1}{2}$ 

$$l = 17$$

1

$$r = 8 \text{ cm}$$

 $\frac{1}{2}$

Total S.A. of remaining solid = C.S.A of cylinder + C.S.A of cone + Area of base

$$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$$

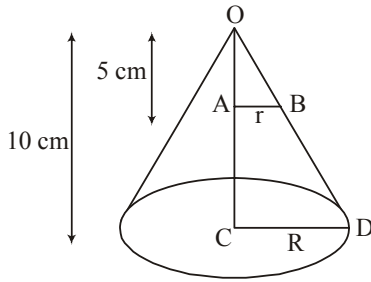
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$$= \pi \times 8(2 \times 15 + 17 + 8) = 8\pi(55) = 440\pi \text{ cm}^2$$

1

OR

The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.



For correct fig 1

$$\Delta OAB \sim \Delta OCD$$

$$\frac{OA}{OC} = \frac{AB}{CD} \Rightarrow \frac{5}{10} = \frac{r}{R}$$

$$\Rightarrow R = 2r$$

1

$$\frac{V \text{ of cone}}{V \text{ of frustum}} = \frac{\frac{1}{3}\pi r^2 5}{\frac{1}{3}\pi(r^2 + R^2 + rR)5} = \frac{r^2}{7r^2} = \frac{1}{7}$$

1+1

or 7 : 1

40. The mode of the following frequency distribution is 36. Find the missing frequency (f).

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	8	10	f	16	12	6	7

Sol. Modal class 30 – 40

$$l = 30 \quad f_0 = f \quad f_1 = 16 \quad f_2 = 12 \quad h = 10$$

1

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$36 = 30 + \frac{16 - f}{32 - f - 12} \times 10$$

2

$$f = 10$$

1

7. The maximum number of zeroes a cubic polynomial can have, is

- (a) 1 (b) 4 (c) 2 (d) 3

Sol. (d) 3

1

8. If α and β are the zeroes of the polynomial $2x^2 - 13x + 6$, then $\alpha + \beta$ is equal to

- (a) -3 (b) 3 (c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

Sol. (c) $\frac{13}{2}$

1

9. The mid-point of the line-segment AB is P(0, 4). If the coordinates of B are (-2, 3) then the coordinates of A are

- (a) (2, 5) (b) (-2, -5) (c) (2, 9) (d) (-2, 11)

Sol. (a) (2, 5)

1

10. In Fig.-1 AP, AQ and BC are tangents to the circle with centre O. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is

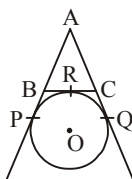


Fig. 1

- (a) 15 (b) 10 (c) 9 (d) 7.5

Sol. (d) 7.5

1

Question numbers 11 to 15, fill in the blanks:

11. The corresponding sides of two similar triangles are in the ratio 3 : 4, then the ratios of the area of triangles is _____.

Sol. 9 : 16

1

12. The area of triangle formed with the origin and the points (4, 0) and (0, 6) is _____.

Sol. 12 sq units

1

OR

The co-ordinate of the point dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 is _____.

Sol. (3, 5)

1

13. The value of $(\tan^2 60^\circ + \sin^2 45^\circ)$ is _____.

Sol. $\frac{7}{2}$ or 3.5

1

14. Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are _____.

Sol. 3 and -2

1

15. The value of $(\sin 43^\circ \cdot \cos 47^\circ + \sin 47^\circ \cos 43^\circ)$ is _____.

Sol. 1

1

Question numbers 16 to 20, answer the following :

16. In figure-2 \widehat{PQ} and \widehat{AB} are two arcs of concentric circles of radii 7 cm and 3.5 cm resp., with centre O. If $\angle POQ = 30^\circ$, then find the area of shaded region.

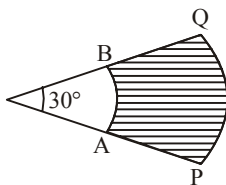


Fig.-2

Sol. Area of shaded region = $\frac{22}{7} \times \frac{30^\circ}{360^\circ} (7^2 - (3.5)^2)$
 $= 9.625 \text{ cm}^2$

 $\frac{1}{2}$ $\frac{1}{2}$

17. If $3k - 2$, $4k - 6$ and $k + 2$ are three consecutive terms of A.P., then find the value of k.

Sol. $(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$

 $\frac{1}{2}$

$\Rightarrow k = 3$

 $\frac{1}{2}$

18. Find the value of $(\cos 48^\circ - \sin 42^\circ)$.

Sol. $\cos 48^\circ - \cos (90^\circ - 42^\circ)$

 $\frac{1}{2}$

$\cos 48^\circ - \cos 48^\circ = 0$

 $\frac{1}{2}$

OR

Evaluate: $(\tan 23^\circ) \times (\tan 67^\circ)$

Sol. $\cos (90^\circ - 67^\circ) \times \tan 67^\circ$

 $\frac{1}{2}$

$= \cot 67^\circ \times \tan 67^\circ = 1$

 $\frac{1}{2}$

19. In a ΔPQR , S and T are points on the sides PQ and PR respectively, such that $ST \parallel QR$. If $PT = 2$ cm and $TR = 4$ cm, find the ratio of the areas of ΔPST and ΔPQR .

Sol.
$$\frac{\text{ar}(\Delta PST)}{\text{ar}(\Delta PQR)} = \left(\frac{PT}{PR}\right)^2 \quad \frac{1}{2}$$

$$= \left(\frac{2}{2+4}\right)^2 = \frac{1}{9}$$

\therefore ratio is 1 : 9

$\frac{1}{2}$

20. Two different coins are tossed simultaneously. What is the probability of getting at least one head?

Sol. Total outcomes = 4 {HH, HT, TH, TT} $\frac{1}{2}$

$P(\text{atleast one head}) = \frac{3}{4}$ $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Prove that: $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Sol. L.H.S = $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$ 1

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} \quad \frac{1}{2}$$

$$= 2\sec^2\theta \quad \frac{1}{2}$$

OR

Prove that: $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Sol. L.H.S = $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$ 1

$$= \cos^2\theta - \sin^2\theta \quad 1$$

22. Divide $(2x^2 - x + 3)$ by $(2 - x)$ and write the quotient and the remainder.

Sol.

$$\begin{array}{r} \overline{) 2x^2 - x + 3} \\ \underline{2x^2 - 4x} \\ + 3 \\ \underline{3x - 6} \\ + 9 \\ \underline{9} \end{array}$$

1

Quotient = $-2x - 3$]

R = 9]

1

23. In a family of three children, find the probability of having at least two boys.

Sol. Total outcomes = 8 {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} 1

$$P(\text{atleast 2 boys}) = \frac{4}{8} \text{ or } \frac{1}{2} \quad 1$$

OR

Two dice are tossed simultaneously. Find the probability of getting

(i) an even number on both dice.

(ii) the sum of two numbers more than 9.

Sol. Total outcomes = 36 cases 1

$$P(\text{even no. on both side}) = \frac{9}{36} \text{ or } \frac{1}{4} \quad \frac{1}{2}$$

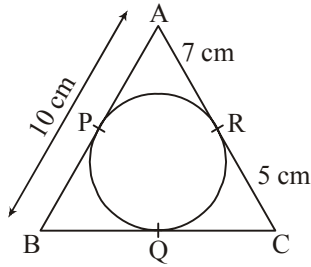
$$P(\text{sum} > 9) = \frac{6}{36} \text{ or } \frac{1}{6} \quad \frac{1}{2}$$

24. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol. Total = 10 + 25 = 35, $P(\text{getting prize}) = \frac{10}{35} \text{ or } \frac{2}{7}$ 1+1

25. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If AB = 10 cm, AR = 7 cm and CR = 5 cm, then find the length of BC.

Sol.



$$AP = AR = 7 \text{ cm}$$

$$PB = 10 - 7 = 3 \text{ cm}$$

$$BQ = BP = 3 \text{ cm}$$

$$QC = RC = 5 \text{ cm}$$

$$BC = 5 + 3 = 8 \text{ cm}$$

 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

26. The length of the minute hand of clock is 14 cm. Find the area swept by the minute hand in 15 minutes.

Sol. Angle swept in 15 minutes = 90°

$$\text{Area} = \frac{22}{7} \times 14 \times 14 \times \frac{90^\circ}{360^\circ}$$

$$= 154 \text{ cm}^2$$

 $\frac{1}{2}$

1

 $\frac{1}{2}$

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. Solve graphically:

$$2x - 3y + 13 = 0; 3x - 2y + 12 = 0$$

Sol. Correct graph of $2x - 3y + 13 = 0$, $3x - 2y + 12 = 0$

$$\text{Solution } x = -2, y = 3$$

1+1

1

28. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ be a rational number

$$\sqrt{3} = \frac{p}{q} \quad p, q \text{ are coprime, } q \neq 0$$

$$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p \quad \text{Let } p = 3m$$

$$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q$$

\therefore 3 is common factor of p and q

Contraction to our assumption

Hence $\sqrt{3}$ is irrational No.

 $\frac{1}{2}$

1

 $\frac{1}{2}$

1

OR

Using Euclid's algorithm, find the HCF of 272 and 1032.

Sol. $1032 = 272 \times 3 + 216$

$$272 = 216 \times 1 + 56$$

$$\frac{1}{2} + \frac{1}{2}$$

$$216 = 56 \times 3 + 48$$

$$56 = 48 \times 1 + 8$$

$$\frac{1}{2} + \frac{1}{2}$$

$$48 = 8 \times 6 + 0$$

$$\text{HCF}(1032, 272) = 8$$

$$\frac{1}{2} + \frac{1}{2}$$

29. If $x = 3 \sin \theta + 4 \cos \theta$ and $y = 3 \cos \theta - 4 \sin \theta$ then prove that $x^2 + y^2 = 25$.

Sol. $x^2 = 9 \sin^2 \theta + 16 \cos^2 \theta + 24 \sin \theta \cos \theta$

1

$$y^2 = 9 \cos^2 \theta + 16 \sin^2 \theta - 24 \sin \theta \cos \theta$$

1

$$x^2 + y^2 = 25$$

1

OR

If $\sin \theta + \sin^2 \theta = 1$; then prove that $\cos^2 \theta + \cos^4 \theta = 1$.

Sol. $\sin \theta = 1 - \sin^2 \theta = \cos^2 \theta$

1

$$\text{L.H.S} = \cos^2 \theta + (\cos^2 \theta)^2 = \cos^2 \theta + \sin^2 \theta$$

1+1

$$= 1 = \text{R.H.S}$$

30. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 3. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

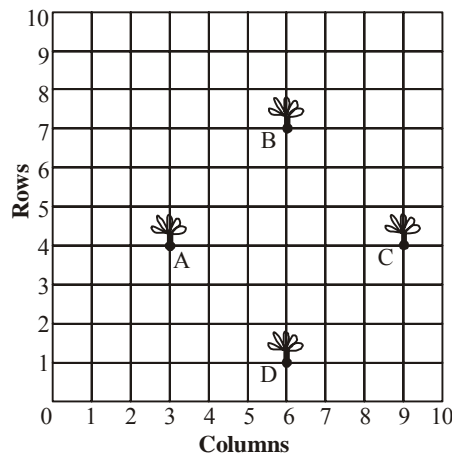


Fig. 3

Sol. A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1)	1
AB = $3\sqrt{2}$, BC = $3\sqrt{2}$, CD = $3\sqrt{2}$, DA = $3\sqrt{2}$	1
AC = 6 unit BD = 6 unit	$\frac{1}{2}$
AB = BC = CD = DA and AC = BD	
ABCD is a square	
\therefore Champa is correct	$\frac{1}{2}$

31. Draw a line segment of length 7 cm and divide it in the ratio 2 : 3.

Sol. Correct construction	3
----------------------------------	---

OR

Draw a circle of radius 4 cm and construct the pair of tangents to the circle from an external point, which is at a distance of 7 cm from its centre.

Sol. Correct construction	3
----------------------------------	---

32. If α and β are the zeroes of the polynomial $f(x) = 5x^2 - 7x + 1$, then find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

Sol. $\alpha + \beta = \frac{7}{5}$ and $\alpha\beta = \frac{1}{5}$	$\frac{1}{2} + \frac{1}{2}$
----------------------------------------------------------------------------	-----------------------------

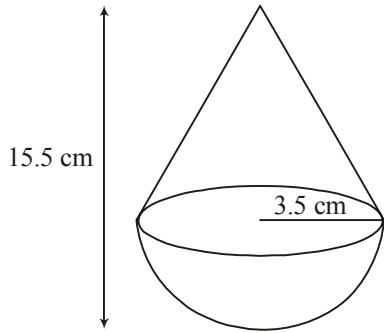
$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{7}{5}\right)^2 - 2 \times \frac{1}{5}}{\frac{1}{5}}$$

$$= \frac{39}{5} \text{ or } 7.8$$

33. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, find the total surface area of the toy.

Sol.



Height of cone = $15.5 - 3.5 = 12$ cm

$\frac{1}{2}$

Slant height, $l = \sqrt{(12)^2 + (3.5)^2} = 12.5$ cm

1

TSA of toy = $\pi(3.5) \times 12 + 2\pi(3.5)^2$

1

= 66.5π or 209 cm²

$\frac{1}{2}$

34. In the Fig.-4, two circles touch each other at a point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

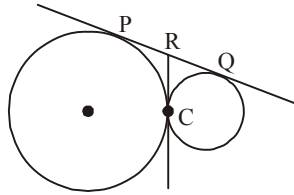


Fig. 4

Sol. $PR = RC$... (1) }
 $PQ = RC$... (2) }

[Tangents from external point]

1

1

From (1) and (2), $PR = PQ$

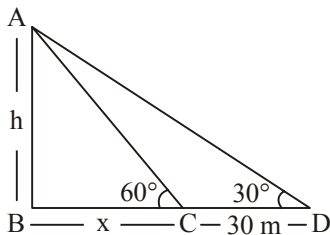
1

SECTION D

Question numbers 35 to 40 carry 4 marks each.

35. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on opposite bank is 60° . When he moves 30 m away from the bank, he finds the angle of elevation to be 30° . Find the height of the tree and width of the river. [Take $\sqrt{3} = 1.732$]

Sol.



Correct figure

1

In right ΔABC

$\tan 60^\circ = \frac{h}{x}$

$\frac{1}{2}$

$\sqrt{3}x = h$... (1)

$\frac{1}{2}$

In rt ΔABD $\tan 30^\circ = \frac{h}{30+x} \Rightarrow \frac{30+x}{\sqrt{3}} = h$... (2)

$\frac{1}{2} + \frac{1}{2}$

Solving (1) & (2) $x = 15$ m, $h = 15\sqrt{3}$ m = 25.98 m

$\frac{1}{2} + \frac{1}{2}$

36. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Sol. Correct Fig., given, to prove, construction $4 \times \frac{1}{2} = 2$
 Correct proof given, to prove, construction, 2

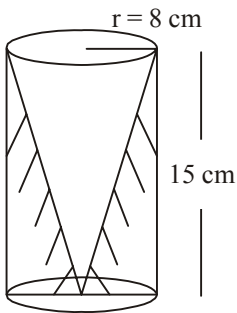
OR

Prove that the length of tangents drawn from an external point to a circle are equal.

Sol. Correct Fig., given, to prove, construction $4 \times \frac{1}{2} = 2$
 Correct proof given, to prove, construction, 2

37. From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of remaining solid. (Give your answer in terms of π).

Sol. Correct figure $\frac{1}{2}$



$$l = 17$$

$$r = 8 \text{ cm}$$

$$\text{Total S.A. of remaining solid} = \text{C.S.A of cylinder} + \text{C.S.A of cone} + \text{Area of base}$$

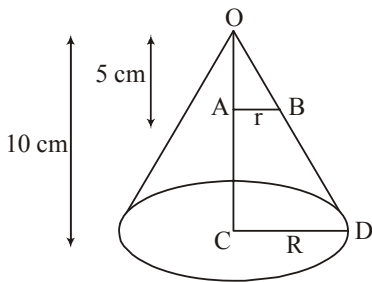
$$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$$

$$= \pi \times 8(2 \times 15 + 17 + 8) = 8\pi(55) = 440\pi \text{ cm}^2$$

OR

The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.

Sol. For correct fig 1



$$\Delta OAB \sim \Delta OCD$$

$$\frac{OA}{OC} = \frac{AB}{CD} \Rightarrow \frac{5}{10} = \frac{r}{R}$$

$$\Rightarrow R = 2r$$

$$\frac{V \text{ of cone}}{V \text{ of frustum}} = \frac{\frac{1}{3}\pi r^2 \cdot 5}{\frac{1}{3}\pi(r^2 + R^2 + rR)} = \frac{r^2}{7r^2} = \frac{1}{7}$$

$$\text{or } 7 : 1$$

38. The 17th term of an A.P. is 5 more than twice its 8th term. If 11th term of A.P. is 43; then find its nth term.

Sol. $a_{17} = 2a_8 + 5 \Rightarrow a + 16d = 2(a + 7d) + 5$ 1

$\Rightarrow 2d - a = 15$... (1)

$a_{11} = 43 \Rightarrow a + 10d = 43$... (2) 1

Solving (1) & (2) $a = 3$ $d = 4$ 1

$a_n = 4n - 1$ 1

OR

How many terms of A.P. 3, 5, 7, 9, ... must be taken to get the sum 120?

Sol. $a = 3, d = 3, S_n = 120$ 1

$\frac{n}{2}[2 \times 3 + (n-1)2] = 120 \Rightarrow n^2 + 2n - 120 = 0$ 1

$(n + 12)(n - 10) = 0$ 1

$n = -12, n = 10$ 1

Reject $n = -12, n = 10$

39. Find the median for the given frequency distribution:

Classes	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75
Frequency	2	3	8	6	6	3	2

Sol.

Class	Frequency	cf
40-50	2	2
45-50	3	5
55-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Correct table 1

Median class = 55 - 60 $\frac{1}{2}$

$$\begin{aligned} \text{Median} &= 55 + \frac{\left(\frac{30}{2} - 13\right)}{6} \times 5 && 2 \\ &= 56\frac{2}{3} \text{ or } 56.67 && \frac{1}{2} \end{aligned}$$

40. If the price of a book is reduced by ₹ 5, a person can buy 4 more books for ₹ 600. Find the original price of the book.

Sol. Let original price of the book be ₹x

A.T.Q.

$$\frac{600}{x-5} - \frac{600}{x} = 4 \quad 1\frac{1}{2}$$

$$x^2 - 5x - 750 = 0 \quad 1$$

$$(x - 30)(x + 25) = 0 \quad 1$$

$$x = 30 \text{ or } -25$$

Price is always positive, so original price of book is ₹30 1/2

QUESTION PAPER CODE 430/2/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option.

1. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?

(a) 2 (b) 4 (c) 5 (d) 1

Sol. (c) 5 1

2. The maximum number of zeroes a cubic polynomial can have, is

(a) 1 (b) 4 (c) 2 (d) 3

Sol. (d) 3 1

3. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then the value of y is

(a) 3 (b) -3 (c) 7 (d) 4

Sol. (a) 3 1

4. Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Sol. (d) $\frac{3}{4}$ 1

5. $2\sqrt{3}$ is

(a) an integer (b) a rational number
(c) an irrational number (d) a whole number

Sol. (c) an irrational no. 1

6. The cumulative frequency table is useful in determining

(a) Mean (b) Median (c) Mode (d) All of these

Sol. (b) Median 1

7. HCF of two numbers is 27 and their LCM is 162. If one of the number is 54, then the other number is

(a) 36 (b) 35 (c) 9 (d) 81

Sol. (d) 81 1

8. x-axis divides the line segment joining A(2, -3) and B(5, 6) in the ratio:

- (a) 2 : 3 (b) 3 : 5 (c) 1 : 2 (d) 2 : 1

Sol. (c) 1 : 2

1

9. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to their product, then k equals.

- (a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Sol. (d) $-\frac{2}{3}$

1

10. A chord of a circle of radius 10 cm, subtends a right angle at its centre. The length of the chord (in cm) is

- (a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$ (c) $10\sqrt{2}$ (d) $10\sqrt{3}$

Sol. (c) $10\sqrt{2}$

1

Question numbers 11 to 15, fill in the blanks:

11. The value of $(\tan^2 60^\circ + \sin^2 45^\circ)$ is _____.

Sol. $\frac{7}{2}$ or 3.5

1

12. The corresponding sides of two similar triangles are in the ratio 3 : 4, then the ratios of the area of triangles is _____.

Sol. 9 : 16

1

13. Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are _____.

Sol. 3 and -2

1

14. The area of triangle formed with the origin and the points (4, 0) and (0, 6) is _____.

Sol. 12 sq units

1

OR

The co-ordinate of the point dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 is _____.

Sol. (3, 5)

1

15. The value of $\frac{\sin \theta}{\cos(90^\circ - \theta)} + \frac{\cos 43^\circ}{\sin 47^\circ}$ is _____

Sol. 2

1

Question numbers 16 to 20, answer the following :

16. If $3k - 2$, $4k - 6$ and $k + 2$ are three consecutive terms of A.P., then find the value of k .

Sol. $(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$

$\frac{1}{2}$

$\Rightarrow k = 3$

$\frac{1}{2}$

17. Find the value of $(\cos 48^\circ - \sin 42^\circ)$.

Sol. $\cos 48^\circ - \cos (90^\circ - 42^\circ)$

$\frac{1}{2}$

$\cos 48^\circ - \cos 48^\circ = 0$

$\frac{1}{2}$

OR

Evaluate: $(\tan 23^\circ) \times (\tan 67^\circ)$

Sol. $\tan (90^\circ - 67^\circ) \times \tan 67^\circ$

$\frac{1}{2}$

$= \cot 67^\circ \times \tan 67^\circ = 1$

$\frac{1}{2}$

18. In figure-2 \widehat{PQ} and \widehat{AB} are two arcs of concentric circles of radii 7 cm and 3.5 cm resp., with centre O. If $\angle POQ = 30^\circ$, then find the area of shaded region.

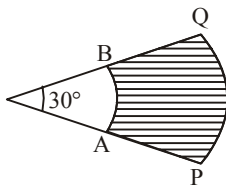


Fig.-2

Sol. Area of shaded region = $\frac{22}{7} \times \frac{30^\circ}{360^\circ} (7^2 - (3.5)^2)$

$\frac{1}{2}$

$= 9.625 \text{ cm}^2$

$\frac{1}{2}$

19. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting a red king.

Sol. $P(\text{Red king}) = \frac{2}{52}$ or $\frac{1}{26}$

1

Sol.
$$\text{L.H.S} = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} = \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} = \cos^2 \theta - \sin^2 \theta$$

23. In a family of three children, find the probability of having at least two boys.

Sol. Total outcomes = 8 {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} 1

$$P(\text{atleast 2 boys}) = \frac{4}{8} \text{ or } \frac{1}{2}$$
 1

OR

Two dice are tossed simultaneously. Find the probability of getting

(i) an even number on both dice.

(ii) the sum of two numbers more than 9.

Sol. Total outcomes = 36 1

$$P(\text{even no. on both side}) = \frac{9}{36} \text{ or } \frac{1}{4}$$
 $\frac{1}{2}$

$$P(\text{sum} > 9) = \frac{6}{36} \text{ or } \frac{1}{6}$$
 $\frac{1}{2}$

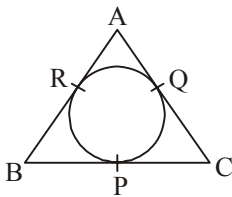
24. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol. Total = 10 + 25 = 35 $P(\text{getting prize}) = \frac{10}{35} \text{ or } \frac{2}{7}$ 1+1

25. An isosceles triangle ABC, with AB = AC, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.

Sol.

$$AB = AC$$



$$AR + RB = AQ + QC$$
 $4 \times \frac{1}{2}$

$$RB = QC \text{ (AR = AQ)}$$

$$BP = PC \Rightarrow P \text{ bisect BC}$$

26. The radius of a circle is 17.5 cm. Find the area of the sector of the circle enclosed by two radii and an arc 44 cm in length.

Sol.
$$\text{Area} = \frac{1}{2}lr = \frac{1}{2} \times 44 \times 17.5 = 385 \text{ cm}^2$$
 1+1

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ be a rational number

$$\sqrt{3} = \frac{p}{q} \quad p, q \text{ are coprime } q \neq 0 \quad \frac{1}{2}$$

$$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p \quad \text{Let } p = 3m \quad 1$$

$$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q \quad \frac{1}{2}$$

\therefore 3 is common factor of p and q

Contraction to our assumption 1

Hence $\sqrt{3}$ is irrational No.

OR

Using Euclid's algorithm, find the HCF of 272 and 1032.

Sol. $1032 = 272 \times 3 + 216$

$$272 = 216 \times 1 + 56 \quad \frac{1}{2} + \frac{1}{2}$$

$$216 = 56 \times 3 + 48$$

$$56 = 48 \times 1 + 8 \quad \frac{1}{2} + \frac{1}{2}$$

$$48 = 8 \times 6 + 0 \quad \text{HCF}(1032, 272) = 8 \quad \frac{1}{2} + \frac{1}{2}$$

28. If $x = 3 \sin \theta + 4 \cos \theta$ and $y = 3 \cos \theta - 4 \sin \theta$ then prove that $x^2 + y^2 = 25$.

Sol. $x^2 = 9 \sin^2 \theta + 16 \cos^2 \theta + 24 \sin \theta \cos \theta \quad 1$

$$y^2 = 9 \cos^2 \theta + 16 \sin^2 \theta - 24 \sin \theta \cos \theta \quad 1$$

$$x^2 + y^2 = 25 \quad 1$$

OR

If $\sin \theta + \sin^2 \theta = 1$; then prove that $\cos^2 \theta + \cos^4 \theta = 1$.

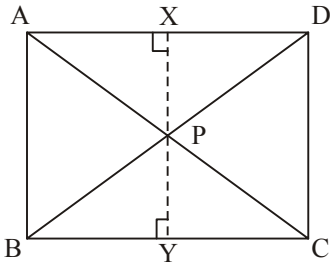
Sol. $\sin \theta = 1 - \sin^2 \theta = \cos^2 \theta \quad 1$

$$\text{L.H.S} = \cos^2 \theta + (\cos^2 \theta)^2 = \cos^2 \theta + \sin^2 \theta \quad 1+1$$

$$= 1 = \text{R.H.S}$$

29. In a rectangle ABCD, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Sol.



Correct figure & Construction

$$\frac{1}{2} + \frac{1}{2}$$

$$\text{In rt } \triangle APX \quad AP^2 = AX^2 + PX^2 \quad \left. \vphantom{\text{In rt } \triangle APX} \right\}$$

$$\text{In rt } \triangle PCY \quad PC^2 = PY^2 + YC^2 \quad \left. \vphantom{\text{In rt } \triangle PCY} \right\} \frac{1}{2}$$

$$\text{In rt } \triangle PBY \quad PB^2 = PY^2 + BY^2 \quad \left. \vphantom{\text{In rt } \triangle PBY} \right\} \frac{1}{2}$$

$$\text{In rt } \triangle PDX \quad PD^2 = DX^2 + PX^2 \quad \left. \vphantom{\text{In rt } \triangle PDX} \right\}$$

$$PA^2 + PC^2 = AX^2 + PX^2 + PY^2 + YC^2 \quad \left. \vphantom{PA^2 + PC^2} \right\}$$

$$= BY^2 + PY^2 + PX^2 + XD^2 \quad \left. \vphantom{= BY^2 + PY^2 + PX^2 + XD^2} \right\} 1$$

$$= PB^2 + PD^2$$

30. Draw a line segment of length 7 cm and divide it in the ratio 2 : 3.

Sol. Correct construction

3

OR

Draw a circle of radius 4 cm and construct the pair of tangents to the circle from an external point, which is at a distance of 7 cm from its centre.

Sol. Correct construction

3

31. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 3. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

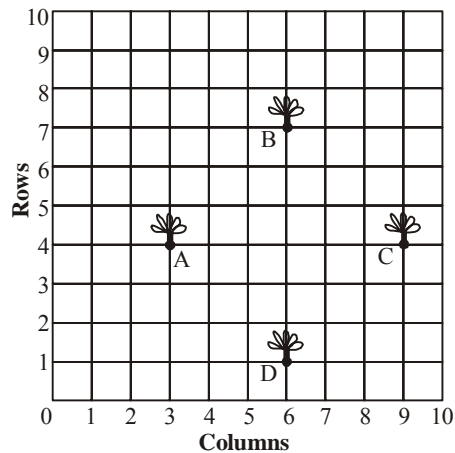


Fig. 3

Sol. $A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1)$	1
$AB = 3\sqrt{2}, BC = 3\sqrt{2}, CD = 3\sqrt{2}, DA = 3\sqrt{2}$	1
$AC = 6 \text{ unit} \quad BD = 6 \text{ unit}$	$\frac{1}{2}$
$AB = BC = CD = DA$ and $AC = BD$	
ABCD is a square	
\therefore Champa is correct	$\frac{1}{2}$

32. Solve graphically:

$$2x - 3y + 13 = 0; \quad 3x - 2y + 12 = 0$$

Sol. Correct graph of $2x - 3y + 13 = 0, 3x - 2y + 12 = 0$	1+1
Solution $x = -2, y = 3$	1

33. A horse is tethered to one corner of a rectangular field of dimensions 70 m \times 52 m, by a rope of length 21 m. How much area of the field can it graze?

Sol. Area of field = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$	1+1
$= 346.5 \text{ cm}^2$	1

34. Find the quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. Hence find the zeroes.

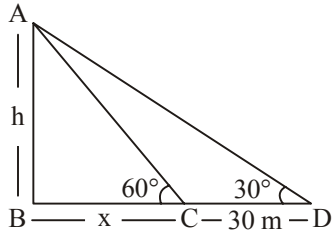
Sol. Polynomial $K(x^2 + 3x + 2)$	
Put $K = 1 \Rightarrow$ required polynomial $x^2 + 3x + 2$	$\frac{1}{2}$
$x^2 + 3x + 2 = (x + 2)(x + 1)$	1
\therefore Zeroes are $-2, -1$	$\frac{1}{2}$

SECTION D

Question numbers 35 to 40 carry 4 marks each.

35. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on opposite bank is 60° . When he moves 30 m away from the bank, he finds the angle of elevation to be 30° . Find the height of the tree and width of the river. [Take $\sqrt{3} = 1.732$]

Sol.



Correct figure

1

In right $\triangle ABC$

$$\tan 60^\circ = \frac{h}{x} \quad \frac{1}{2}$$

$$\sqrt{3}x = h \quad \dots(1) \quad \frac{1}{2}$$

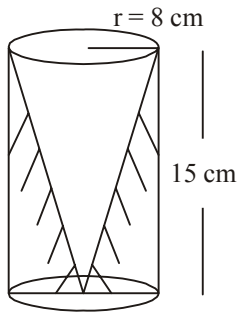
$$\text{In rt } \triangle ABD \tan 30^\circ = \frac{h}{30+x} \Rightarrow \frac{30+x}{\sqrt{3}} = h \quad \dots(2) \quad \frac{1}{2} + \frac{1}{2}$$

$$\text{Solving (1) \& (2) } x = 15\text{m, } h = 15\sqrt{3} \text{ m} = 25.98 \text{ m} \quad \frac{1}{2} + \frac{1}{2}$$

36. From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of remaining solid. (Give your answer in terms of π).

Sol.

Correct figure $\frac{1}{2}$



$$l = 17$$

1

$$r = 8 \text{ cm}$$

$\frac{1}{2}$

Total S.A. of remaining solid = C.S.A of cylinder + C.S.A of cone + Area of base

$$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$$

1

$$= \pi \times 8(2 \times 15 + 17 + 8) = 8\pi(55) = 440\pi \text{ cm}^2$$

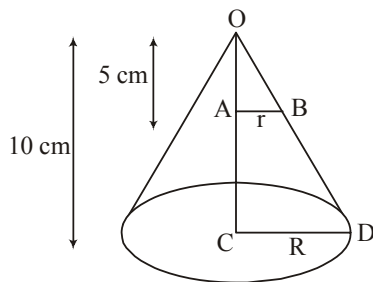
1

OR

- The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.

Sol.

For correct fig 1



$$\triangle OAB \sim \triangle OCD$$

$$\frac{OA}{OC} = \frac{AB}{CD} \Rightarrow \frac{5}{10} = \frac{r}{R}$$

$$\Rightarrow R = 2r$$

1

$$\frac{\text{V of cone}}{\text{V of frustum}} = \frac{\frac{1}{3}\pi r^2 5}{\frac{1}{3}\pi(r^2 + R^2 + rR)} = \frac{r^2}{7r^2} = \frac{1}{7}$$

1+1

or 7 : 1

37. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Sol. Correct Fig., given, to prove, construction $4 \times \frac{1}{2} = 2$
 Correct proof given, to prove, construction, 2

OR

Prove that the length of tangents drawn from an external point to a circle are equal.

Sol. Correct Fig., given, to prove, construction $4 \times \frac{1}{2} = 2$
 Correct proof given, to prove, construction, 2

38. The 17th term of an A.P. is 5 more than twice its 8th term. If 11th term of A.P. is 43; then find its nth term.

Sol. $a_{17} = 2a_8 + 5 \Rightarrow a + 16d = 2(a + 7d) + 5$ 1
 $\Rightarrow 2d - a = 15$... (1)
 $a_{11} = 43 \Rightarrow a + 10d = 43$... (2) 1
 Solving (1) & (2) $a = 3$ $d = 4$ 1
 $a_n = 4n - 1$ 1

OR

How many terms of A.P. 3, 5, 7, 9, ... must be taken to get the sum 120?

Sol. $a = 3, d = 3, S_n = 120$ 1
 $\frac{n}{2}[2 \times 3 + (n-1)2] = 120 \Rightarrow n^2 + 2n - 120 = 0$ 1
 $(n + 12)(n - 10) = 0$ 1
 $n = -12, n = 10$ 1
 Reject $n = -12, n = 10$

39. Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46. Find the integers.

Sol. Let three consecutive +ve integers $x, x + 1, x + 2$
 $x^2 + (x + 1)(x + 2) = 46$ 1
 $2x^2 + 3x - 44 = 0 \Rightarrow 2x^2 + 11x - 8x - 44 = 0$ 1
 $\Rightarrow (2x + 11)(x - 4) = 0$

$$\Rightarrow x = \frac{-11}{2}, x = 4 \quad 1$$

\Rightarrow 3 consecutive integers are 4, 5, 6 1

40. Find the mean of the following distribution:

Class	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	2	3	7	6	6	6

Sol.	C.I.	x_i	f	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
	10-25	17.5	2	-2	-4
	25-40	32.5	3	-1	-3
	40-55	47.5 a	7	0	0
	55-70	62.5	6	1	6
	70-85	77.5	6	2	12
	85-100	92.5	6	3	18
			<u>30</u>		<u>29</u>

Correct Table 2

$$\text{Mean} = 47.5 + \frac{29}{30} \times 15 \quad 1$$

$$= 47.5 + 14.5 = 62 \quad 1$$
