

# SOF INTERNATIONAL MATHEMATICS OLYMPIAD 2022-23

### DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

#### Guidelines for the Candidate

- 1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
- 2. Write your Name, School Code, Class, Roll No. and Mobile Number clearly on the OMR Sheet and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
- 3. The Question Paper comprises four sections :
  - Section 1: Logical Reasoning (15 Questions)
  - Section 2 : Mathematical Reasoning (20 Questions) or Applied Mathematics (20 Questions)
  - Section 3 : Everyday Mathematics (10 Questions)
  - Section 4 : Achievers Section (5 Questions)
- 4. Section-1, 3 and 4 are compulsory for all. In Section-2 opt for Mathematical Reasoning OR Applied Mathematics and mark the same on the OMR Sheet.

Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.

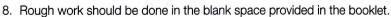
- 5. All questions are compulsory. There is no negative marking. Use of calculator is not permitted.
- 6. There is only ONE correct answer. Choose only ONE option for an answer.
- To mark your choice of answers by darkening the circles on the OMR Sheet, use HB Pencil or Blue / Black ball point pen only. E.g. Q.16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is \_\_\_\_\_.

A. 11.450 kg B. 11.000 kg

C. 11.350 kg

D. 11.250 kg

As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet.



- 9. Return the OMR Sheet to the invigilator at the end of the exam.
- 10. Please fill in your personal details in the space provided on this page before attempting the paper.



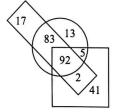


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## LOGICAL REASONING

1. Study the given Venn diagram carefully and answer the following question.



Which number represents women soldiers who are punctual?

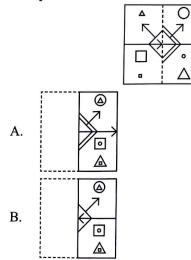
- A. 2
- B. 17
- C. 92
- D. 83
- 2. Some digits are coded as shown below:

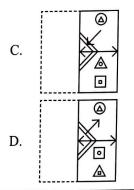
Digits	7	3	5	0	2	1	6	4	9	8
Codes	N	Н	L	Т	F	D	R	Q	G	Р

While coding the given number following conditions are also to be observed.

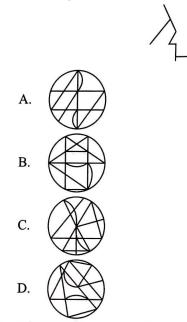
#### **Conditions:**

- (i) If the first digit is even and the last digit is odd, then they are to be coded as \$ and @ respectively.
- (ii) 0 is not considered as either even or odd.
- (iii) If 0 is preceded as well as followed by an odd digit, then 0 is to be coded as ↑.
- (iv) If 0 is preceded as well as followed by an even digit, then 0 is to be coded as  $\downarrow$ .
- Find the code for 4670510.
- A. QRN<sup>1</sup>LDT
- B. \$RN↑LD@
- C. QRN↓DLT
- D.  $RN\downarrow LD@$
- 3. A square transparent sheet with a pattern and a dotted line on it is shown here. Select a figure from the options as to how the pattern would appear when the transparent sheet is folded along the dotted line.



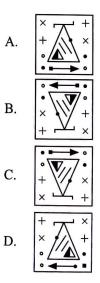


4. In which of the following options, the given figure is exactly embedded as one of its parts?



5. Choose the correct mirror image of the given figure, if mirror is placed vertically to the left.



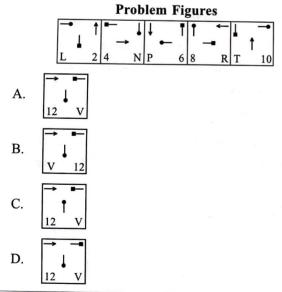


6. Find the missing numbers in the given number series.

- 1, 1, 4, 8, 9, 27, 16, <u>?</u>, <u>?</u>
- A. 36, 49
- B. 64, 25
- C. 64, 125
- D. 36, 81
- 7. How many numbers are there in the given arrangement, each of which is immediately preceded by a prime number and immediately followed by an odd number?

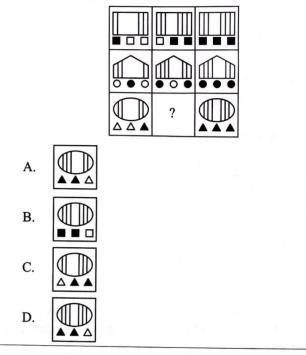
2 6 9 7 9 5 4 3 4 3 8 6 7 5 3 1 7 9 6 1 8 2 3 9

- A. 4
- B. 5
- C. 6
- D. More than 6
- 8. If 'A \$ B' means 'A is brother of B', 'A @ B' means 'A is wife of B', 'A # B' means 'A is daughter of B' and 'A \* B' means 'A is father of B', then which of the following expressions indicate the relationship 'U is father-in-law of P'?
  - A. P@Q\$T#U\*W
  - B. P@W\$Q\*T#U
  - C. P@Q\$W\*T#U
  - D. P@Q\$T#W\*U
- 9. Select a figure from the options which will continue the same series as established by the Problem Figures.

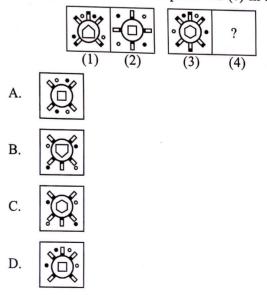


10. Point B is 12 m South of point A. Point C is 24 m East of point B. Point D is 8 m South of point C. Point D is 12 m East of Point E and Point F is 8 m North of Point E. If a man is standing facing North at Point C, then how far and in which direction is point F from him?

- A. 12 m, East
- B. 24 m, West
- C. 12 m, West
- D. 24 m, East
- 11. Select a figure from the options which will complete the given figure matrix.

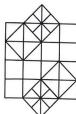


- 12. Select the odd one out.
  - A. LUNQ
  - В. НРЈМ
  - C. QZSV
  - D. DMFI
- 13. There is a certain relationship between figures (1) and (2). Establish a similar relationship between figures (3) and (4) by selecting a suitable figure from the given options that would replace the (?) in fig. (4).



IMO | Class-11 | Set-A | Level 1 | S PF

14. Find the number of squares formed in the given figure.



- A. 30
- B. 34
- C. 40

11

TC

D. More than 40 15. A, B, C, D, E, F, G, H and K are sitting around a circle facing the centre. F is fourth to the right of A who is third to the right of B. K is fourth to the left of B and third to the right of D. C is third to the right of H. E is second to the left of G.

Who is fourth to the left of G?

- С A.
- Β. A
- C. D
- D. K

0 A. B.

1

 $\frac{-1}{15}$ 

 $\frac{1}{15}$ 

C.

D.

## MATHEMATICAL REASONING

10.	II ta	an $A +$	$\sin A = m$ and $\tan A - \sin A = n$ , then	
	$(m^2)$	$(-n^2)^2$	is equal to	
		mn	is equal to	
	A.	4		
	В.	3		

- C. 16
- D. 9
- 17. If p and q are positive integers such that  $p = a^2 b^2$ and  $q = a^3 b^3$ , where a, b are prime numbers, then HCF (p, q) =
  - A. ab
  - $a^2b^2$ B.
  - $a^3b^2$ C.
  - D.  $a^3b^3$
- 18. If (1 + 2i)(2 + 3i)(3 + 4i) = x + iy, then find the value of  $x^{2} + y^{2}$ .
  - A. 1625
  - B. 875
  - C. 1652
  - D. 925

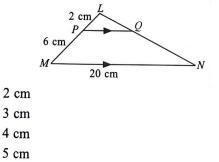
A.

B.

C.

D.

19. In  $\Delta LMN$ , PQ || MN such that LP = 2 cm and PM = 6 cm. If MN = 20 cm, then find PQ.



Evaluate:  $\lim_{x \to 2} \left[ \frac{2x^2 - 9x + 10}{5x^2 - 5x - 10} \right]$ 20.

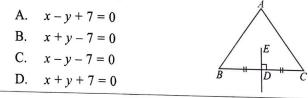
21. Read the given statements carefully and select the correct option.

Statement-I : The area of the quadrant of a circle having circumference of 44 cm is  $\frac{77}{2}$  cm<sup>2</sup>.

Statement-II : The area of a sector of a circle of

radius r with central angle x° is  $\frac{x}{360^{\circ}} \times \pi r^2$ .

- Both Statement-I and Statement-II are false. A.
- B. Both Statement-I and Statement-II are true.
- C. Statement-I is true but Statement-II is false.
- Statement-I is false but Statement-II is true. D.
- 22. Find the equation of the perpendicular bisector drawn to the side BC of  $\triangle ABC$ , whose vertices are A(-2, 1), B(2, 3) and C(4, 5).



23. The equation of the parabola whose vertex is at (2, -1) and focus at (2, -3) is

A. 
$$x^2 + 4x - 8y - 12 = 0$$

B.  $x^2 - 4x + 8y + 12 = 0$ 

C. 
$$x^2 + 8y = 12$$

D.

$$x^2 - 4x + 12 = 0$$

- 24. If one zero of the polynomial  $f(x) = (k^2 + 4)x^2 + 13x + 4k$  is the reciprocal of the other, then k is equal to
  - A. 2 B. -2
  - C. 1
  - D. 1
- 25. The values of x for which  $-11 \le 4x 3 \le 13$ , is
  - A.  $-4 \le x \le 5$ B.  $-2 \le x \le 4$ C.  $-8 \le x \le 16$
  - D.  $-11 \le x \le 10$
- 26. Solve for  $x: \frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ , where  $a, b, x \neq 0$ and  $a + b + x \neq 0$ . A. x = a or x = bB. x = -a or x = b
  - C. x = a or x = -b
  - D. x = -a or x = -b
- 27. If n(A) = 10, n(B) = 6 and n(C) = 5 for three disjoint sets A, B, C then  $n(A \cup B \cup C) =$ 
  - A. 21
  - B. 11
  - C. 1
  - D. 9
- 28. Find the mode of the following data :

1216	Marks	Frequency 7			
-	0-10				
	10-20	14			
	20-30	13			
	30-40	12			
ade	40-50	20			
1. 52	50-60	11			
	60-70	15			
	70-80	8			
A.	44.7				
B.	43.8				
C.	42.5				
D.	38.6				

- 29. The constant term in the expansion of  $\left(x^2 \frac{1}{x^2}\right)^{10}$  is
  - A.  ${}^{16}C_8$
  - B.  ${}^{16}C_7$

- C.  ${}^{16}C_9$
- D.  ${}^{16}C_{10}$
- 30. Find the derivative of  $(x^2 + 1) \cos x$ .
  - A.  $2x \cos x (x^2 + 1) \sin x$
  - B.  $2x \sin x x^2 \cos x$
  - C.  $x^2(\cos x \sin x)$
  - D.  $2x(\sin x + \cos x)$
- 31. The sum of the first term and the fifth term of an ascending A.P. is 26 and the product of the second term with the fourth term is 160. Find the sum of the first seven terms of this A.P.
  - A. 120
  - B. 112
  - C. 210
  - D. 512
- 32. The points (1, 2, 3), (-1, -1, -1) and (3, 5, 7) are
  - A. Vertices of an equilateral triangle
  - B. Vertices of a scalene triangle
  - C. Vertices of a right triangle
  - D. Collinear.
- 33. How many 5-letter words, with or without meaning, can be formed out of the letters of the word 'EQUATIONS', if repetition of letters is not allowed?
  - A. 126
  - B. 5<sup>9</sup>
  - C. 9<sup>5</sup>
  - D. 15120
- 34. A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble at random from the jar is 1/3, and the probability of selecting a green marble at random is 4/9. How many white marbles does the jar contain?
  - A. 10
  - B. 12
  - C. 15
  - D. 18
- 35. If  $A = \{x : x \in W, x < 2\}, B = \{x : x \in N, 1 < x < 5\}, C = \{3, 5\}$ , then find  $A \times (B \cap C)$ .
  - A.  $\{(0, 1), (0, 5)\}$
  - B.  $\{(1, 3), (1, 5)\}$
  - C. {(1, 3), (1, 5), (0, 3), (0, 5)}
  - D.  $\{(0, 3), (1, 3)\}$

5

# APPLIED MATHEMATICS

OR

- 16. A man borrowed a sum of money and agrees to pay off by paying ₹ 3150 at the end of the first year and ₹ 4410 at the end of the second year. If the rate of compound interest is 5% per annum, then find the sum borrowed.
  - A. ₹ 7000
  - B. ₹6500
  - C. ₹7800
  - D. None of these
- 17. Find the values of p and q for which the following system has infinitely many solutions.

2x + 3y = 7(p + q)x + (2p - q)y = 21 A. p = 4, q = 3 B. p = 1, q = 5 C. p = 5, q = 1 D. p = 3, q = 4

- 18. The price of a bicycle is ₹ 3136 inclusive of tax (under GST), at the rate of 12% on its listed price. A buyer asks for a discount on the listed price so that after charging GST, the selling price becomes equal to the listed price. Find the amount of discount which the seller has to allow for the deal.
  - A. ₹2800
  - B. ₹300
  - C. ₹ 336
  - D. None of these
- 19. The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. (Use  $\pi = 22/7$ )
  - A.  $9.84 \text{ cm}^2$
  - B.  $10.27 \text{ cm}^2$
  - C.  $510 \text{ cm}^2$
  - D.  $616 \text{ cm}^2$
- 20. Find the number of arrangements of the letters of the word INDEPENDENCE when words begin with I and end in P.
  - A. 12400
  - B. 12420
  - C. 12600
  - D. 12620

21. If the line segment joining the points (3, -4) and (1, 2) is trisected at points P(a, -2) and  $Q\left(\frac{5}{3}, b\right)$ . Then,

A. 
$$a = \frac{8}{3}, b = \frac{2}{3}$$
  
B.  $a = \frac{7}{3}, b = 0$   
C.  $a = \frac{1}{3}, b = 1$   
D.  $a = \frac{2}{3}, b = \frac{1}{3}$ 

- 22. If y = (x + 1)(x + 2)(x + 3)(x + 4)(x + 5), then the value of  $\frac{dy}{dx}$  at x = 0 is equal to A. 374 B. 742 C. 472 D. 274
- 23. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many like both coffee and tea?
  - A. 15B. 16
  - C. 18
  - D. 19
- 24. If  $\alpha$  and  $\beta$  are the zeroes of the quadratic polynomial  $p(x) = x^2 + x + 2$ , then find a polynomial whose zeroes are  $2\alpha 1$  and  $2\beta 1$ .
  - A.  $x^2 + 4x + 11$
  - B.  $x^2 4x 11$
  - C.  $x^2 + 4x 11$
  - D.  $x^2 4x + 11$
- 25. Find the domain of the function  $f(x) = \sqrt{x^2 5}$ .
  - A.  $R (-\sqrt{5}, \sqrt{5})$ B. RC. [0, 1]
  - D. (0,∞)

26. The median of the distribution given below is 14.4. Find the values of x and y, if the sum of the frequencies is 20.

Class-interval	Frequency		
0-6	4		
6-12	x		
12-18	5		
18-24	У		
24-30	1		
A. $x = 4, y = 6$			
B. $x = 6, y = 4$			
C. $x = 5, y = 5$			

- D. x = 2, y = 8
- 27. If coordinates of point A are (5, -3) and B is a point on the X-axis such that the slope of line AB is -2, then coordinates of B are

A. 
$$(7, 2)$$
  
B.  $\left(\frac{7}{2}, 0\right)$   
C.  $\left(0, \frac{7}{2}\right)$   
D.  $\left(\frac{2}{7}, 0\right)$ 

- 28. If  $\log 7 \log 2 + \log 16 2 \log 3 \log \frac{7}{45} = 1 + \log n$ , then find *n*.
  - A. 3
  - B. 4
  - C. 2
  - D. 5
- 29. The sum of first three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find the three terms.
  - A. 1, 7, 13
  - B. 1, 6, 14
  - C. 2, 8, 11
  - D. None of these
- Mr Dutt lives in Mumbai and consumes 84 SCM gas in 60 days. Calculate his bill amount.

Units of consumption (in SCM)	Price per unit (in ₹)
up to 0.60 SCM/day	₹ 29.60
0.61-1.50 SCM/day	₹ 35.20

A.	₹ 3909.80
B.	₹ 4102.60
C.	₹ 3601.70

- D. ₹ 3099.60
- 31. A metal cube of 9 cm edge is melted and recast into three smaller cubes. If the edge of two of the smaller cubes are 1 cm and 6 cm. Find the edge of the third cube.
  - A. 5 cm
  - B. 8 cm
  - C. 10 cm
  - D. 12 cm
- 32. If coefficient of correlation between two variables X and Y is 0.25, their covariance is 25 and var (X) is 16, then the standard deviation of Y-series is
  - A. 25
  - B. 2.5
  - C. 0.25 D. 0.0025
- 33.  $\frac{5\sin^2 30^\circ + \cos^2 45^\circ 4\tan^2 30^\circ}{2\sin 30^\circ \cos 30^\circ + \tan 45^\circ}$  is equal to A.  $\frac{5}{6}(2+\sqrt{3})$ B.  $\frac{2-\sqrt{3}}{2+\sqrt{3}}$ C.  $\frac{5}{6}$ D.  $\frac{5}{6}(2-\sqrt{3})$ 
  - 34. A set of *n* variates  $x_1, x_2, ..., x_n$  has means  $\overline{x}$  and standard deviation  $\sigma$ . The mean and standard deviation

of *n* values  $\frac{x_1}{k}, \frac{x_2}{k}, \dots, \frac{x_n}{k}$   $(k \neq 0)$  respectively are

- A.  $k\overline{x}, \frac{\sigma}{k}$ B.  $\frac{\overline{x}}{k}, \frac{\sigma}{k}$ C.  $k\overline{x}, k\sigma$ D.  $\frac{\overline{x}}{k}, k\sigma$
- 35. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows an even number". Then, I and F are
  - A. Mutually exclusive events
  - B. Exhaustive events
  - C. Mutually exclusive and exhaustive events
  - D. None of these

- 36. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.
  - A.  $418 \text{ cm}^2$
  - B.  $572 \text{ cm}^2$
  - C.  $726 \text{ cm}^2$
  - D.  $1000 \text{ cm}^2$
- 37. Cards numbered from 1 to 64 are placed in a box. A card is drawn at random from the box. Find the probability that the number on the card drawn is a perfect cube.

A.	$\frac{1}{16}$	
B.	$\frac{3}{16}$	
C.	$\frac{5}{16}$	
D.	$\frac{9}{16}$	

- 38. Two trains of lengths 200 m and 225 m are running in the opposite direction with respective speed of 35 km/h and 55 km/h. Find the time when they cross one another.
  - A. 18 secs
  - B. 16 secs
  - C. 15 secs
  - D. 17 secs
- 39. A person opens an account with ₹ 50 and starts depositing every day double the amount he has deposited on the previous day. Find the amount he has deposited on the 10<sup>th</sup> day from the beginning.
  - A. ₹12800
  - B. ₹25600
  - C. ₹ 50200
  - D. ₹28600
- 40. A fruit vendor purchased 15 dozen oranges at ₹ 56 per dozen. He sold 10 dozen at 12% profit and rest 5 dozen at 30% profit. What is the profit percentage in the whole transaction?
  - A. 42%
  - B. 18%
  - C. 25%
  - D. 21%
- 41. There are 96 apples and 112 oranges. These fruits are packed in boxes in such a way that each box contains

fruits of the same variety, and every box contains an equal number of fruits. Find the minimum number of boxes in which all the fruits can be packed.

- A. 12B. 13
- C. 14
- D. 15
- 42. Three pipes A, B, C can fill a cistern in 8 hours, 10 hours and 15 hours respectively. Find the time taken by all the three pipes to fill the tank when pipes working together.
  - A.  $3\frac{3}{7}$  hours B.  $5\frac{3}{7}$  hours C.  $4\frac{3}{7}$  hours D.  $8\frac{3}{7}$  hours
- 43. The average age of wife, husband and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of husband is
  - A. 50 years
  - B. 40 years
  - C. 30 years
  - D. None of these
- 44. In a class test, the sum of the marks obtained by Ankur in Mathematics and Science is 28. If he had got 3 more marks in Mathematics and 4 marks less in Science, then product of marks obtained in the two subjects would have been 180. Which of the following can be the marks obtained in the two subjects separately?
  - A. 12, 16
  - B. 8, 20
  - C. 15, 13
  - D. 21, 7
- 45. A group of 27 persons is assumed to complete the work in 36 days. After 30 days, 9 more persons were employed and they complete the work in 3 days before time. In how many days would it have been finished if no extra persons were employed?
  - A. 32 days
  - B. 29 days
  - C. 2 days
  - D. None of these

- 46. Solve the following:
  - (i) Find two positive numbers whose difference is 12 and whose A.M. exceeds the G.M. by 2.
  - (ii) Find the 15<sup>th</sup> term of the sequence defined by

$$a_n = \frac{2n}{3n-1}.$$
(i) (ii)  
A. 15, 6 16/27  
B. 16, 4 15/22  
C. 12, 8 15/22  
D. 16, 4 17/25

47. Read the given statements carefully and select the correct option.

**Statement-I**: The number of ways in which 5 boys and 5 girls can sit in a circle so that no two boys sit together is 9!.

**Statement-II**: A committee of 3 persons is to be constituted from a group of 2 men and 3 women. The number of ways in which this can be done is 10.

- A. Both Statement-I and Statement-II are true.
- B. Both Statement-I and Statement-II are false.
- C. Statement-I is true but Statement-II is false.
- D. Statement-I is false but Statement-II is true.
- 48. Two dice are thrown. The events A, B and C are as follows :
  - A: getting an even number on the first die.
  - B: getting an odd number on the first die.

C : getting the sum of the numbers on the dice  $\leq 5$ .

Then, which of the following is true?

- (i) A and B are mutually exclusive.
- (ii) A and B are mutually exclusive and exhaustive.
- (iii) A = B'
- (iv) A and C are mutually exclusive.
- (v) A and B' are mutually exclusive.
- (vi) A', B' and C are mutually exclusive and exhaustive.

- A. (i) only
- B. (ii), (iv) and (v) only
- C. (iii) and (vi) only
- D. (i), (ii) and (iii) only
- 49. Fill in the blanks and select the correct option.
  - (i) The derivative of  $\frac{(x+5)(2x^2-1)}{x}$  is \_\_\_\_\_. (ii) If  $y = \frac{2x^9}{3} - \frac{5}{7}x^7 + 6x^3 - x$ , then  $\frac{dy}{dx}$  at x = 1 is equal to \_\_\_\_\_. (i) (ii) A.  $4x - 10 - \frac{5}{x^2}$  10 B.  $4x + 10 - \frac{5}{x^2}$  18 C.  $4x + 10 + \frac{5}{x^2}$  18 D.  $-4x + 10 + \frac{5}{x^2}$  10
- 50. Read the given statements carefully and state T for true and F for false.
  - (i) If  $A = \{8, 9, 10\}$ ,  $B = \{1, 2, 3, 4, 5\}$ , then the number of elements in  $A \times A \times B$  is 45.
  - (ii) Let  $A = \{1, 2, 3, 4\}$  and R be the relation on A defined by  $\{(a, b) : a, b \in A, a \times b \text{ is an even number}\}$ , then the range of R is  $\{2, 3, 4\}$ .
  - (iii) If n(A) = m, then the number of relations in A are  $2^{m}$ .

	(i)	(ii)	(iii)
A.	Т	F	Т
В.	F	Т	F
C.	Т	F	F
D.	F	Т	Т

SPACE FOR ROUGH WORK