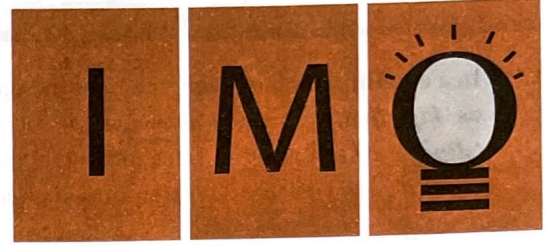


CLASS
12

QUESTION
PAPER SET

A



**SOF INTERNATIONAL
MATHEMATICS OLYMPIAD
2022-23**

DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Guidelines for the Candidate

- You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
- Write your **Name, School Code, Class, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
- The Question Paper comprises four sections :
Section - 1: **Logical Reasoning** (15 Questions)
Section - 2 : **Mathematical Reasoning** (20 Questions) or **Applied Mathematics** (20 Questions)
Section - 3 : **Everyday Mathematics** (10 Questions)
Section - 4 : **Achievers Section** (5 Questions)
- Section-1, 3 and 4 are compulsory for all.** In Section-2 opt for Mathematical Reasoning OR Applied Mathematics and mark the same on the OMR Sheet.
Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
- All questions are compulsory. There is no negative marking. Use of calculator is not permitted.
- There is only ONE correct answer. Choose only ONE option for an answer.
- To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.
Q.16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is _____.
A. 11.450 kg B. 11.000 kg C. 11.350 kg D. 11.250 kg
As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet.
- Rough work should be done in the blank space provided in the booklet.
- Return the OMR Sheet to the invigilator at the end of the exam.
- Please fill in your personal details in the space provided on this page before attempting the paper.

16. ● (B) (C) (D)



Inspiring Young Minds Through Knowledge Olympiads

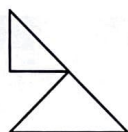


Name:.....

SOF Olympiad Roll No.:..... Contact No.:.....

1. In a certain code language, 'he likes mango' is written as 'kip sip dip', 'money likes mango alot' is written as 'dip kip tif nit' and 'he likes money, is written as 'tif sip dip'. How is 'alot' written in that code language?
- tif
 - sip
 - nit
 - kip

2. Select a figure from the options in which the given figure is exactly embedded as one of its parts.



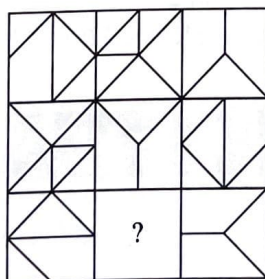
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3. Find the missing numbers in the given number series.

12, 10, 17, 15, 23, 21, 30, ?, ?

- 28, 38
- 28, 37
- 27, 38
- 27, 37

4. Select a figure from the options which will complete the given figure matrix.



-
-
-
-

5. The given question consists of three statements followed by three conclusions numbered I, II and III. Read all the conclusions and find which of the given conclusions logically follows from the given statements, if all statements are assumed to be true.

Statements:

- All lions are animals.
- No bird is an animal.
- Some birds are humans.

Conclusions:

- Some animals are lions.
 - All humans are animals.
 - Some birds are lions.
- Both I and III follow
 - Only I follows
 - Both II and III follow
 - None of these

6. In the given question, two rows of numbers are given. The resultant number in each row is to be worked out separately based on the following rules. The operations on the number progress from left to right.

Rules:

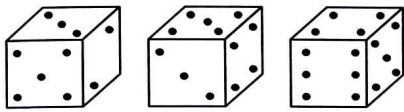
- If a two digit even number is followed by a prime number, then the second number is to be subtracted from the first number.
- If a prime number is followed by an odd number, then both the numbers are multiplied.
- If a perfect cube is followed by a perfect square, then both the numbers are added.
- If an odd number is followed by an even number, then square root of both the numbers are multiplied with each other.

- (v) If an even number is followed by another even number, then the first number is to be divided by the second number.

27 25 4
26 T 15

If T is the resultant of the first row, then what is the resultant of the second row?

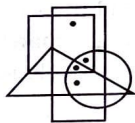
- A. 195
B. 13
C. 180
D. 28
7. Three positions of a dice are shown below. Find the number of dots on the face opposite to the face having six dots.



- A. 2
B. 1
C. 4
D. 3
8. Select the correct water image of the given combination of letters, numbers and symbols.

15@UG\$19F%

- A. 12@NC21dE%
B. %7RI2GU@21
C. %J61\$GN@5I
D. 15@UG21dE%
9. Select a figure from the options which satisfies the same conditions of placement of the dots as in the given figure.



- A.
- B.
- C.

D.

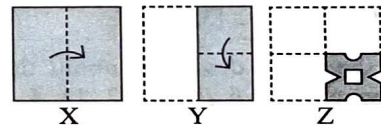


10. If 'P' stands for '+', 'Q' stands for '×', 'R' stands for '-' and 'S' stands for '÷', then which of the following options becomes incorrect?
- A. (4 Q 72) S 12 R 16 P 20 = 28
B. (14 P 11) Q 10 S 5 P 52 R 21 = 80
C. 16 P 9 Q (21 R 11) S 2 = 61
D. 4 P (21 P 7) S 7 Q 9 = 40

11. Read the following information carefully and answer the question that follows:
- 'P × Q' means 'P is brother of Q'.
'P + Q' means 'P is father of Q'.
'P - Q' means 'P is sister of Q'.
'P ÷ Q' means 'P is mother of Q'.

Which of the following represents 'M is the nephew of N'?

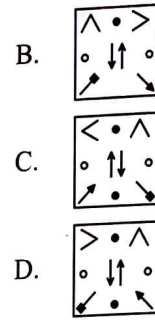
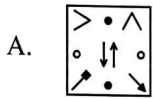
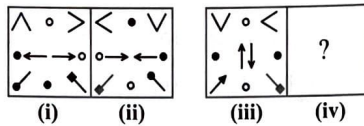
- A. $N - K + M$
B. $N \times K \div M$
C. $N \div K \times M$
D. $N - K + M \times T$
12. The given question consists of a set of three figures X, Y and Z showing a sequence of folding of a piece of paper. Fig. Z shows the manner in which the folded paper has been cut. Select a figure from the options which would most closely resemble the unfolded form of fig. Z.



- A.
- B.
- C.
- D.

13. How many such pairs of letters are there in the word SATURATION each of which has as many letters between them in the word as in the English alphabets?
- A. Two
B. Three
C. Four
D. More than four

14. There is a certain relationship between figures (i) and (ii). Establish the similar relationship between figures (iii) and (iv) by selecting a suitable figure from the options that would replace (?) in figure (iv).



15. Six people – C, D, E, F, G and H are standing in a straight line facing north not necessarily in the same order. D is standing second to the right of F. C is standing fourth to the left of H and H is not standing on the extreme end of the line. E is standing second to the right of D.

Who is standing second to the right of C?

- A. F
B. D
C. G
D. E

MATHEMATICAL REASONING

16. If A and B are matrices of same order, then $(AB' - BA')$ is a

- A. Skew-symmetric matrix
B. Null matrix
C. Symmetric matrix
D. Unit matrix.

17. The interval in which the function $f(x) = x^3 - 6x^2 + 9x + 10$ is strictly increasing is,

- A. $(-\infty, 1) \cup (3, \infty)$
B. $[1, 3]$
C. $(-\infty, 1] \cup [3, \infty)$
D. $(-\infty, -1] \cup [3, \infty)$

18. Find the mean deviation about the mean for the following data : 6, 7, 10, 12, 13, 4, 8, 12.

- A. 2.25
B. 2.50
C. 2.75
D. 2.60

19. The objective function $z = 4x_1 + 5x_2$, subject to $2x_1 + x_2 \geq 7$, $2x_1 + 3x_2 \leq 15$, $x_2 \leq 3$, $x_1, x_2 \geq 0$ has minimum value

- A. At a point on x -axis
B. At a point on y -axis
C. At the origin
D. On the line parallel to x -axis.

20. The value of $\frac{[(\sqrt{2} + i\sqrt{3}) + (\sqrt{2} - i\sqrt{3})]}{[(\sqrt{3} + i\sqrt{2}) + (\sqrt{3} - i\sqrt{2})]}$ =

- A. $\frac{2}{3} + 3i$
B. $\frac{\sqrt{2}}{3} + 5i$
C. $\frac{\sqrt{3}}{2} + 2i$
D. $\frac{\sqrt{2}}{3} + 0i$

21. Let S be the set of all real numbers. A relation R has been defined on S by $aRb \Leftrightarrow |a - b| \leq 1$, then R is

- A. Symmetric and transitive but not reflexive
B. Reflexive and transitive but not symmetric
C. Reflexive and symmetric but not transitive
D. An equivalence relation.

22. Find the equation of the line through the point $(-1, 2, 3)$ which is perpendicular to the lines

$$\frac{x}{2} = \frac{y-1}{-3} = \frac{z+2}{-2} \quad \text{and} \quad \frac{x+3}{-1} = \frac{y+2}{2} = \frac{z-1}{3}$$

- A. $\frac{x-2}{2} = \frac{y+5}{4} = \frac{z+7}{-1}$
B. $\frac{x+4}{3} = \frac{y+2}{-5} = \frac{z+3}{6}$
C. $\frac{x+1}{3} = \frac{y+6}{2} = \frac{z-5}{-2}$
D. $\frac{x+1}{5} = \frac{y-2}{4} = \frac{z-3}{-1}$

23. Solve the inequality $\frac{1}{2}\left(\frac{3}{5}x+4\right) \geq \frac{1}{3}(x-6)$.

- A. $(0, \infty]$
- B. $(-20, 0]$
- C. $(-\infty, \infty]$
- D. $(-\infty, 120]$

24. The area of the region bounded by the parabola $y^2 = x$ and the straight line $2y = x$ is

- A. $\frac{4}{3}$ sq. units
- B. 1 sq. unit
- C. $\frac{2}{3}$ sq. unit
- D. $\frac{1}{3}$ sq. unit

25. An urn contains 9 balls, 2 of which are white, 3 blue and 4 black. 3 balls are drawn at random from the urn. The chance that 2 balls will be of the same colour and the third of a different colour is

- A. $\frac{45}{84}$
- B. $\frac{55}{84}$
- C. $\frac{35}{84}$
- D. $\frac{25}{84}$

26. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x} dx =$

- A. $-\log(1 + \sin^2 x) + C$
- B. $\log(1 + \cos^2 x) + C$
- C. $-\log(1 + \cos^2 x) + C$
- D. $\log(1 + \tan^2 x) + C$

27. Let $f: R \rightarrow R$ be defined by $f(x) = x^2 - \frac{x^2}{1+x^2}$ for all $x \in R$. Then

- A. f is one - one but not onto
- B. f is onto but not one - one
- C. f is both one - one and onto
- D. f is neither one - one nor onto.

28. Out of 6 gentlemen and 4 ladies a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady in each committee?

- A. 246
- B. 240
- C. 244
- D. 242

29. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

- A. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$
- B. $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
- C. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
- D. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

30. The value of $4 \sin \alpha \sin\left(\alpha + \frac{\pi}{3}\right) \sin\left(\alpha + \frac{2\pi}{3}\right) =$

- A. $\sin 3\alpha$
- B. $\sin 2\alpha$
- C. $\sin \alpha$
- D. $\sin^2 \alpha$

31. For the differential equation $x \frac{dy}{dx} + 2y = xy \frac{dy}{dx}$,

- A. Order is 1 and degree is 1
- B. Solution is $\ln(yx^2) = C - y$
- C. Order is 1 and degree is 2
- D. Solution is $\ln(xy^2) = C + y$.

32. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, then find $\frac{dy}{dx}$.

- A. $x^{\sin x - \cos x} \left[\frac{\sin x - \cos x}{x} \right] + x$
- B. $x^{\sin x - \cos x} + \frac{2x}{(x^2 + 1)^2}$
- C. $x^{\sin x - \cos x} \left[\frac{\sin x - \cos x}{x} + (\cos x + \sin x) \log x \right] + \frac{4x}{(x^2 + 1)^2}$
- D. None of these

33. If e_1 is the eccentricity of the conic $9x^2 + 4y^2 = 36$ and e_2 is the eccentricity of the conic $9x^2 - 4y^2 = 36$ then which of the following is true?

- A. $e_1^2 + e_2^2 = 2$
- B. $3 < e_1^2 + e_2^2 < 4$
- C. $e_1^2 + e_2^2 > 4$
- D. None of these

34. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

- A. $[1, 2]$
- B. $[2, 3]$
- C. $[2, 3)$
- D. $[1, 2)$

35. Let R be a relation on the set of natural numbers N defined by $R = \{(x, y) : x + 2y = 8\}$, then find the range of R .

- A. $\{1, 2\}$
 B. $\{2, 4\}$
 C. $\{1, 2, 3\}$
 D. $\{1, 2, 4\}$

OR

APPLIED MATHEMATICS

16. Which of the following binary numbers is equivalent to decimal number 24?

- A. 1101111
 B. 11000
 C. 111111
 D. 11001

17. The order of the single matrix obtained from

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix} \right\} \text{ is}$$

- A. 2×3
 B. 2×2
 C. 3×2
 D. 3×3

18. If $x \equiv 4 \pmod{7}$, then positive values of x are

- A. $\{4, 11, 18, \dots\}$
 B. $\{11, 18, 25, \dots\}$
 C. $\{4, 8, 12, \dots\}$
 D. $\{1, 8, 15, \dots\}$

19. In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. If each student has taken either Mathematics or Economics or both, then the number of students who have taken Economics but not Mathematics is

- A. 7
 B. 25
 C. 18
 D. 32

20. The probability distribution of a discrete random variable X is given below :

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

Find the value of k .

- A. 30
 B. 32
 C. 28
 D. 34

21. In a certain factory turning out razor blades, there is a small chance $\frac{1}{500}$ for any blade to be defective. The blades are in packets of 10. Use Poisson's distribution to calculate the approximate number of packets containing no defective blades in a consignment of 10000 packets. (Use $e^{-0.02} = 0.9802$)

- A. 9802
 B. 9602
 C. 2
 D. None of these

22. There are 10 men and 4 women. What is the probability that committee of 3 persons is to be formed such that it will contain more women than men?

- A. $\frac{12}{41}$
 B. $\frac{18}{57}$
 C. $\frac{14}{51}$

- D. None of these

23. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$. Find the value of a .

- A. 100
 B. 120
 C. 140
 D. 160

24. A 95% confidence interval for a population mean was reported to be 152 to 160. If $\sigma = 15$, what sample size was used in this study? (Given $Z_{0.025} = 1.96$)

- A. 54
 B. 52
 C. 50
 D. 48

25. The test marks of 15 students out of 10 are given below:

Test marks	1	2	3	4	5	6	7	8	9	10
No. of students	1	2	1	1	2	2	5	0	1	0

Find the percentile rank of test marks 6.

- A. 53.33
 B. 52.33
 C. 54.33
 D. 56.33

26. The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units, find the radius of the balloon after t seconds.

- A. $t(2t + 1)$
 B. $8(8t + 3)$
 C. $\sqrt[3]{9(7t + 3)}$
 D. $\sqrt[3]{6(6t + 3)}$

27. Consider the following data:

Years	Production
1983	137
1984	140
1985	134
1986	137
1987	151
1988	121
1989	124
1990	159
1991	157
1992	196
1993	172
1994	150

Calculate a suitable 3-yearly moving average and select the correct option.

A.

Years	3-yearly moving average
1983	—
1984	137
1985	137
1986	140.67
1987	136.33
1988	132
1989	134.67
1990	146.67
1991	170.67
1992	175
1993	172.67
1994	—

B.

Years	3-yearly moving average
1983	—
1984	411
1985	411
1986	422
1987	409
1988	396
1989	404
1990	440
1991	485
1992	498
1993	491
1994	—

C.

Years	3-yearly moving average
1983	—
1984	137
1985	139
1986	141
1987	138
1988	130
1989	135
1990	147
1991	165
1992	168
1993	163.67
1994	—

- D. None of these

28. If the nominal rate is 12 % compounded monthly, then find the corresponding effective interest rate per annum. (Given: $(1.01)^{12} = 1.13$)

- A. 10 %
 B. 13 %
 C. 15 %
 D. None of these

29. A firm anticipates an expenditure of ₹ 500000 for plant modernization at end of 10 years from now. How much should the company deposit at the end of each year into a sinking fund earning interest 5% per annum? (Given: $(1.05)^{10} = 1.629$)

- A. ₹ 39745.63
 B. ₹ 37945.36
 C. ₹ 73954.71
 D. ₹ 43975.63

30. The sum of the A.M. and G.M. of two distinct positive numbers is equal to the difference between the numbers. The numbers are in the ratio
- 1 : 3
 - 1 : 6
 - 9 : 1
 - 1 : 12

31. Aman invested ₹ 20000 in a mutual fund in year 2010. The value of mutual fund increased to ₹ 32000 in year 2015. Calculate the compound annual growth rate of his investment. (Given: $(1.6)^{1/5} = 1.098$)
- 10 %
 - 9 %
 - 9.5 %
 - 9.8 %

32. A boat covers a certain distance downstream in 1 hour while it comes back in $1\frac{1}{2}$ hours. If the speed of the stream be 3 km/h, what is the speed of boat in still water?
- 12 km/h
 - 13 km/h
 - 14 km/h
 - 15 km/h

33. A retailer purchase a fan for ₹ 1500 from a wholesaler and sells it to a consumer at 10% profit. If the sales are intra-state and the rate of GST is 12%, then the tax (under GST) received by the Central Government is
- ₹ 18
 - ₹ 198
 - ₹ 90
 - ₹ 99

34. Evaluate: $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^{2x} dx.$
- $\frac{e}{2}$
 - $\frac{7}{2}$
 - $\frac{e^2}{2} \left(\frac{e^2}{2} - 1 \right)$
 - None of these

35. Let $S(k) : 1 + 3 + 5 + \dots + (2k - 1) = 3 + k^2$. Then which of the following is true?
- $S(k) \Rightarrow S(k + 2)$
 - $S(k) \Rightarrow S(k + 1)$
 - $S(k) \Rightarrow S(k + 3)$
 - All of these

EVERYDAY MATHEMATICS

36. A person earns 15% on an investment but loses 12% on another investment. If the ratio of the two investments be 3:5, then what is gain or loss percent in the whole transaction?

- $6\frac{1}{4}$ % loss
- $13\frac{1}{8}$ % gain
- $13\frac{1}{8}$ % loss
- $1\frac{7}{8}$ % loss

37. If 8 men working 9 hours a day can build a wall 18 m long, 2 m broad and 12 m high in 10 days, how many men will be required to build a wall 32 m long, 3 m broad and 9 m high by working 6 hours a day, in 8 days?

- 30
- 28
- 25
- 32

38. A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Find the probability that it is either red or blue.

- $\frac{2}{9}$
- $\frac{7}{9}$
- $\frac{1}{9}$
- $\frac{4}{9}$

39. Two years ago, father was three times as old as his son and two years hence, twice his age will be equal to five times that of his son. Find the present age of father.

- 36 years
- 40 years
- 38 years
- 34 years

40. A and B started a business jointly. A 's investment was thrice the investment of B and period of his investment was twice the period of investment of B . If B gets ₹ 9000 as profit, then what will be the 25% of total profit?

- A. ₹ 1575
- B. ₹ 2250
- C. ₹ 63000
- D. ₹ 15750

41. Average of 9 observations was found to be 35. Later on, it was detected that an observation 81 was misread as 18. Find the correct average.

- A. 42
- B. 40
- C. 44
- D. 38

42. A drum contains a mixture of two liquids A and B in the ratio 5 : 3. When 6 litres of mixture are drawn off and the drum is filled with B , the ratio of A and B becomes 15 : 17. How many litres of liquid B was contained in the drum initially?

- A. 9 litres
- B. 15 litres
- C. 12 litres
- D. 8 litres

43. 20 persons are invited for a party. In how many different ways can they and the host be seated at circular table, if the two particular persons are to be seated on either side of the host?

- A. 20!
- B. $2 \times 18!$
- C. 18!
- D. None of these

44. Taps A , B and C fills a tank in 8 hours. After $\frac{2}{3}$ hours, tap C is closed and A and B fill the remaining tank. Find the time in which the tank can be filled by tap C alone, if A and B fills the remaining part in 11 hours.

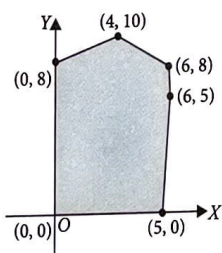
- A. 12 hours
- B. 24 hours
- C. 20 hours
- D. 15 hours

45. Monica has a piece of canvas whose area is 552 m^2 . She uses it to make a conical tent with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 2 m^2 , find the volume of the tent that can be made with it.

- A. 1225 m^3
- B. 1200 m^3
- C. 1230 m^3
- D. 1232 m^3

ACHIEVERS SECTION

46. The feasible region for a LPP is shown shaded in the figure. Let $Z = 3x - 4y$ be the objective function.



- (i) Minimum of Z occurs at
- (ii) Maximum of Z occurs at

- | | (i) | (ii) |
|----|--------|--------|
| A. | (5, 0) | (6, 5) |
| B. | (0, 8) | (5, 0) |
| C. | (6, 5) | (5, 0) |
| D. | (0, 8) | (6, 5) |

47. Read the given statements carefully and select the correct option.

Statement-I : The value of $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$ is $5x^2 + 8x$.

Statement-II : The cofactors of a_{21} and a_{31} of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$ are -16 and -8 respectively.

- A. Statement-I is true but Statement-II is false.
- B. Statement-I is false but Statement-II is true.
- C. Both Statement-I and Statement-II are true.
- D. Both Statement-I and Statement-II are false.

48. Match the following and select the correct option.

	Column-I	Column-II
(P)	$y = \frac{a}{x} + b$	(1) $y'' - y = 0$
(Q)	$y = ae^x + be^{-x}$	(2) $y'' - 7y' + 12y = 0$
(R)	$y = ae^{4x} + be^{3x}$	(3) $xy'' + 2y' = 0$
	(P)	(Q)
A.	(2)	(3)
B.	(2)	(1)
C.	(1)	(3)
D.	(3)	(1)

49. Four defective oranges are accidentally mixed with sixteen good ones. Three oranges are drawn from the mixed lot. Find the probability distribution of the number of defective oranges.

A.

X	0	1	2	3
$P(X)$	$3/29$	$4/19$	$8/105$	$3/378$

B.

X	0	1	2	3
$P(X)$	$28/57$	$8/19$	$8/95$	$1/285$

C.

X	0	1	2	3
$P(X)$	$3/19$	$4/95$	$3/25$	$4/285$

D.

X	0	1	2	3
$P(X)$	$5/21$	$4/69$	$7/29$	$9/991$

50. Solve the following:

- (i) The function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$ is strictly increasing for all x , if
- (ii) The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval

- | | | |
|----|-------------------|----------------|
| | (i) | (ii) |
| A. | $k > \frac{3}{2}$ | $(-\infty, 1)$ |
| B. | $k < \frac{3}{2}$ | $(1, \infty)$ |
| C. | $k < \frac{3}{2}$ | $(-\infty, 1)$ |
| D. | $k > \frac{3}{2}$ | $(1, \infty)$ |

SPACE FOR ROUGH WORK