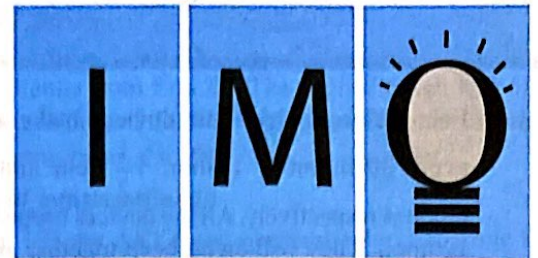


CLASS
10

LEVEL
2



**SOF INTERNATIONAL
MATHEMATICS OLYMPIAD
2019-20**

DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
3. In the school code column in the OMR Sheet, please fill in code allocated to your school and not the exam center code.
4. The Question Paper comprises two sections : **Mathematics** Section (45 Questions) and **Achievers Section** (5 Questions).
Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
5. All questions are compulsory. There is no negative marking. Use of calculator / smart phone is not permitted.
6. There is only ONE correct answer. Choose only ONE option for an answer.
7. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.
Q. 16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is _____.
A. 11.450 kg B. 11.000 kg C. 11.350 kg D. 11.250 kg
As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet. 16. ● (B) (C) (D)
8. Rough work should be done in the blank space provided in this booklet.
9. Please fill in your personal details in the space provided on this page before attempting the paper.
10. **RETURN THE OMR SHEET AND QUESTION PAPER TO THE INVIGILATOR AT THE END OF THE EXAM.**



SCIENCE OLYMPIAD FOUNDATION
Inspiring Young Minds Through Knowledge Olympiads

Name:.....

Section:..... SOF Olympiad Roll No.:..... Contact No.:.....

1. Four different electronic devices make a beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at _____.
- A. 12 midnight
 B. 3 a.m.
 C. 6 a.m.
 D. 9 a.m.

2. If the points $P(x, y)$, $Q(-2, 1)$ and $R(4, -1)$ are collinear and $x - y = 1$, then find the sum of x and y .
- A. 1
 B. -1
 C. 0
 D. 2

3. If one of the roots of $2x^2 + ax + 32 = 0$ is twice the other root, then the value of a is _____.
- A. $-3\sqrt{2}$
 B. $8\sqrt{2}$
 C. $12\sqrt{2}$
 D. $-2\sqrt{2}$

4. Two dice are thrown simultaneously. Find the probability of getting a multiple of 3 as the sum.
- A. $\frac{1}{3}$
 B. $\frac{1}{2}$
 C. $\frac{1}{6}$
 D. $\frac{1}{4}$

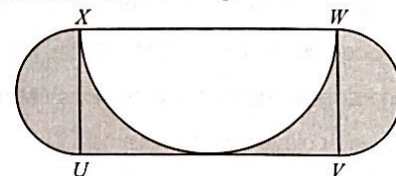
5. A person standing on the top of a multi storeyed building which is 200 m high. He observes the angle of elevation of the top of a tower as 60° and the angle of depression of the base of the tower as 45° . Find the height of the tower (Ignore the height of the person).
- A. 300 m
 B. 546.4 m
 C. 173.2 m
 D. 346.4 m

6. The sum of first p terms of an A.P. is given by $(p^2 + 16p)$. Find the sum of 6th term and 12th term of the A.P.
- A. 66
 B. 36
 C. 76
 D. 82

7. The mean marks scored by 45 students is 350. It was detected later that marks 253 of one student was wrongly copied as 235 for the computation of mean. Find the correct mean.
- A. 335
 B. 353.6
 C. 349
 D. 350.4

8. Find the value of p and q respectively.
- $$\left(\frac{1+m}{m}\right)p + \left(\frac{1+n}{n}\right)q = m-n, \frac{p}{m} - \frac{4q}{n} = 5; mn \neq 0$$
- A. $-n, m$
 B. m^2, n^2
 C. n^2, m^2
 D. $m, -n$

9. In the given figure, $UVWX$ is a rectangle with $UV = 28$ cm and $VW = 14$ cm. XW , VW and UX are the diameters of the three semicircles drawn. Find the shaded area of the figure.

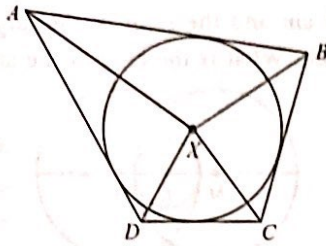


- A. 308 cm^2
 B. 283 cm^2
 C. 238 cm^2
 D. 392 cm^2
10. How many cylindrical iron rods with diameter 1.4 cm and length 4.2 cm are to be melted to form a right circular cone of height 70 cm and diameter 8.4 cm?
- A. 435
 B. 231
 C. 200
 D. 520

11. Solve for y :
- $$pqy^2 + (q^2 - pr)y - qr = 0$$
- A. $\frac{r}{q}, \frac{-q}{r}$
 B. $\frac{-q}{p}, \frac{q}{r}$
 C. $\frac{-q}{p}, \frac{-r}{q}$
 D. $\frac{r}{q}, \frac{-q}{p}$

12. In the given figure, if X is the centre of the circle and $\angle AXB = 120^\circ$, then what is the measure of

$$\angle DXC + \frac{1}{2}(\angle DAB + \angle CBA)?$$



- A. 90°
 B. 60°
 C. 120°
 D. 180°
-
13. The areas of two similar triangles PQR and XYZ are 361 cm^2 and 576 cm^2 respectively. If the median of ΔPQR is 68.4 cm , then find the corresponding median of ΔXYZ .
- A. 86.4 cm
 B. 84.8 cm
 C. 85 cm
 D. 82.1 cm
-
14. How many even numbers beginning with 98 must be taken for their sum to be 2904?
- A. 24
 B. 25
 C. 23
 D. 22
-
15. If $mny^2 = (y + 1)(m - n)^2$, then find the value of $\frac{2^2}{y} + \frac{2^2}{y^2} + 1$.
- A. $\left(\frac{m-n}{m+n}\right)^2$
 B. $\left(\frac{m+n}{m-n}\right)^2$
 C. $\left(\frac{m}{m+n}\right)^2$
 D. $\left(\frac{n}{m+n}\right)^2$
-
16. The shape of a shuttle cock is frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm , the height of the entire shuttle cock is 7 cm . Find its external surface area. (Take $\sqrt{17} = 4.12$)
- A. 82.45 cm^2
 B. 74.26 cm^2
 C. 72.42 cm^2
 D. 86.42 cm^2

17. Let XYZ be a right angled triangle in which $XY = 5 \text{ cm}$, $YZ = 6 \text{ cm}$ and $\angle Y = 90^\circ$. YM is the perpendicular from Y on XZ . The circle through Y, Z, M is drawn. Which of the following steps of construction of tangents from X to this circle is incorrect?

Steps of constructions :

Step-1 : Draw ΔXYZ and perpendicular YM from Y on XZ .

Step-2 : Draw a circle with YZ as a diameter. This circle will pass through M .

Step-3 : Let O be the mid point of YZ . Join XO .

Step-4 : Draw a circle with XO as radius. This circle cuts the circle drawn in Step-2 at Y and P .

Step-5 : Join XP . XP and XY are desired tangents drawn from X to the circle passing through Y, Z and M .

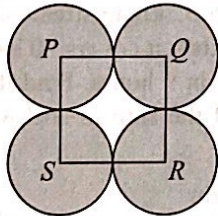
- A. Only Step-1
 B. Only Step-4
 C. Only Step-3
 D. Only Step-5
-
18. ABC is a right angle triangle, right angled at B . If BD is the length of the perpendicular drawn from B to AC . Which of the following is correct?
- (i) $\Delta ADB \sim \Delta ABC$
 (ii) $\Delta BDC \sim \Delta ABC$
 (iii) $BD^2 = AD \times DC$
- A. Only (i)
 B. Only (ii) and (iii)
 C. Only (i) and (iii)
 D. (i), (ii) and (iii)
-
19. A ship covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the difference between the speed of the ship in still water and speed of the stream.
- A. 10 km/hr
 B. 2 km/hr
 C. 12 km/hr
 D. 8 km/hr
-
20. Which of the following rational numbers have terminating decimal expansion?
- (i) $\frac{501}{25}$ (ii) $\frac{621}{25}$
 (iii) $\frac{6805}{27 \times 5^2}$ (iv) $\frac{7105}{7 \times 5^2}$
- A. Only (i) and (iii)
 B. Only (iii) and (iv)
 C. Only (i), (ii) and (iv)
 D. (i), (ii), (iii) and (iv)

21. What must be added to $P(y) = 4y^4 + 2y^3 - 2y^2 + y - 1$ so that the resulting polynomial is divisible by $g(y) = y^2 + 2y - 3$?
- A. $61y - 65$
 B. $22y + 17$
 C. $-61y + 65$
 D. $-22y - 17$

22. If the quadratic equation $(1 + n^2)x^2 + (2nl)x + (l^2 - b^2) = 0$ has equal roots, then $b^2(1 + n^2)$ equals
- A. $-l^2$
 B. $-l + 1$
 C. $l - 1$
 D. l^2

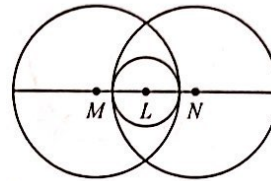
23. Two circles with centres X and Y of radii 5 cm and 12 cm respectively intersect at two points L and M such that XL and YM are tangents to the two circles. Find the length of the common chord LM .
- A. 5.76 cm
 B. 3.6 cm
 C. 9.23 cm
 D. 8.42 cm

24. In the given figure, $PQRS$ is a square and points P, Q, R and S are the centres of four circles each having a radius of length 4 units. If a point is selected at random from the interior of square $PQRS$, then what is the probability that the point will be selected from the unshaded region?



- A. $7/15$
 B. $8/19$
 C. $3/14$
 D. $5/7$
25. A right circular cone is 4.1 cm high and the radius of its base is 2.1 cm. Another right circular cone is 4.3 cm high and the radius of the base is 2.1 cm. Both the cones are melted and recast into a sphere. Find the diameter of the sphere.
- A. 6.4 cm
 B. 4.2 cm
 C. 2.1 cm
 D. 5.6 cm

26. There are two circles intersecting each other. Another smaller circle with centre L , is lying between the common region of two larger circles. Centres of the circle (i.e., M, L and N) are lying on a straight line. $MN = 16$ cm and the radii of the larger circles are 10 cm each. What is the area of the smaller circle?



- A. $4\pi \text{ cm}^2$
 B. $2\pi \text{ cm}^2$
 C. $\frac{4}{\pi} \text{ cm}^2$
 D. $\frac{\pi}{4} \text{ cm}^2$

27. For what value of k will the following pair of linear equations have infinitely many solutions?

$$2x - 3y = 7 \text{ and } (k + 2)x - (2k + 1)y = 3(2k - 1)$$

- A. 1
 B. -4
 C. 2
 D. 4

28. Find the value of x and y respectively in the following frequency distribution table, if the median is 32.5.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	3	5	x	12	6	y	2	40

- A. 9, 6
 B. 10, 15
 C. 15, 13
 D. 9, 3

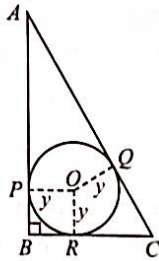
29. Read the following statements carefully and select the correct option.

Statement-I : If the point (m, n) is equidistant from the points $(x + y, y - x)$ and $(x - y, x + y)$, then $my = nx$.

Statement-II : In order to divide a line segment internally in the ratio $a : b$, both a and b are always real numbers.

- A. Statement-I is true but Statement-II is false.
 B. Both Statement-I and Statement-II are false.
 C. Both Statement-I and Statement-II are true.
 D. Statement-I is false but Statement-II is true.

30. $\triangle ABC$ is a right-angled triangle with $AB = 8$ cm and $AC = 10$ cm. A circle with centre O has been inscribed inside the triangle. Find the value of y .



- A. 2 cm
- B. 4 cm
- C. 3 cm
- D. 5 cm

31. If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then the sum of first mn terms is _____.

- A. $mn + 1$
- B. $\frac{mn+1}{2}$
- C. $\frac{mn-1}{2}$
- D. $\frac{mn-1}{3}$

32. A flag staff standing on a horizontal plane is leaning towards East. At two points situated at distances m and n exactly from the foot of tower due West on it, the angles of elevation of the top of the flag staff are respectively α and β . Find the height of the top of the flag staff from the ground.

- A. $\frac{\tan \beta - \tan \alpha}{m - n}$
- B. $\frac{(m - n) \tan \alpha \tan \beta}{\tan \beta - \tan \alpha}$
- C. $(m - n) (\tan \alpha + 2 \tan \beta)$
- D. $\frac{mn(\tan \alpha + \tan \beta)}{\tan \beta - \tan \alpha}$

33. A bag contains 5 green balls and some white balls. If the probability of drawing a white ball is double that of a green ball, then find the number of white balls in the bag.

- A. 22
- B. 33
- C. 10
- D. 0

34. If α and β are the zeroes of the quadratic equation $x^2 - 2x - 3 = 0$, then find the constant term of the quadratic equation whose zeroes are $\frac{\alpha+1}{\alpha-1}$ and $\frac{\beta+1}{\beta-1}$.

- A. 1
- B. 0

- C. -1
- D. 2

35. If the point $Z(a, 2/3)$ divides internally the line segment joining the points $X(-5, -4)$ and $Y(-2, 3)$ in the ratio $m : 1$, then find the value of a .

- A. -8
- B. $\frac{67}{8}$
- C. $\frac{67}{3}$
- D. -3

36. Which of the following options is correct?

- A. If $\tan A = \cot B$, then $A + B = 180^\circ$.
- B. The value of the expression $\sin 90^\circ - \cos 90^\circ$ is negative.
- C. The product of $\cot \theta$ and $\tan \theta$ gives 1.
- D. The value of the expression $\cos^2 73^\circ - \sin^2 27^\circ$ is zero.

37. A straight road leads to the foot of a building. A man standing at the top of the building observes a bus at point P at an angle of depression of 30° , which is approaching the foot of the building with a uniform speed. After 12 seconds, the angle of depression of the bus at point Q is found to be 60° . Find the time taken by the bus to reach the foot of the building from point Q .

- A. 12 seconds
- B. 6 seconds
- C. 9 seconds
- D. 5 seconds

38. The interior of a building is in the form of a cylinder of diameter 4.3 m and 3.8 m height, surmounted by a cone whose vertical angle is a right angle. Find the surface area of the building.

- A. 71.91 m^2
- B. 82.45 m^2
- C. 68.41 m^2
- D. 74.35 m^2

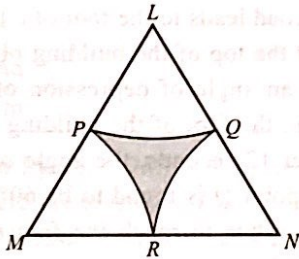
39. X and Y are points on sides AB and AC respectively of $\triangle ABC$. If $AX = 3$ cm, $XB = 6$ cm, $AY = 5$ cm and $YC = 10$ cm, then $BC =$ _____.

- A. $2XY$
- B. $3XY$
- C. XY
- D. $\frac{1}{2}XY$

40. In ΔPQR right angled at Q , $QR = 20$ units and $PR - PQ = 5$ units. Evaluate $\frac{1 + \sin R}{\cos R}$.
- A. 6
B. 8
C. 4
D. 12

41. To divide a line segment PQ in the ratio $6 : 8$, first a ray PX is drawn, so that $\angle QPX$ is an acute angle and then at equal distances points are marked on the ray PX . Find the minimum number of these points.
- A. 2
B. 8
C. 14
D. 12

42. In the given figure, arcs are drawn by taking vertices L, M and N of an equilateral triangle of side 20 cm to intersect the sides MN, NL and LM at their respective mid-points R, Q and P . Find the area of the shaded region. [Use $\pi = 3.14$]



- A. 16.2 cm^2
B. 48.5 cm^2
C. 78.5 cm^2
D. 18.2 cm^2

43. An arc of a circle is of length 5π cm and the sector bounded by the same arc has an area of $20\pi \text{ cm}^2$. Find the radius of the circle.
- A. 6 cm
B. 8 cm
C. 7 cm
D. 5 cm

44. Euclid's division lemma states that for two positive integers x and y , there exist unique integers q and r such that $x = yq + r$, where r must satisfy
- A. $1 < r < y$
B. $0 < r \leq y$
C. $0 \leq r < y$
D. $0 < r < y$

45. If $\sec \theta - \cos \theta = b$ and $\cot \theta + \tan \theta = a$, then which of the following is correct?
- A. $(a^2b)^{1/3} + (ab^2)^{1/3} = 1$
B. $a^2 + b^2 = 2$
C. $(a^2b)^{2/3} - (ab^2)^{2/3} = 1$
D. $(a+b)^2 = \sqrt{2}$

ACHIEVERS SECTION

46. Arrange the given steps in correct order while constructing a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{4}$ of the corresponding sides of the triangle PQR (i.e., of scale factor $\frac{3}{4}$).

1. Locate 4 points Q_1, Q_2, Q_3 and Q_4 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4$.
2. Draw a line through R' parallel to the line RP to intersect QP at P' .
3. Draw any ray QX making an acute angle with QR on the side opposite to the vertex P .
4. Join Q_4R and draw a line through Q_3 parallel to Q_4R to intersect QR at R' .

Then, $\Delta P'QR'$ is the required triangle.

- A. 3, 4, 2, 1
B. 2, 4, 3, 1
C. 3, 1, 4, 2
D. 2, 4, 1, 3

47. Fill in the blanks and select the correct option.
- (i) If the n^{th} term of an A.P. is $5 - 11n$, then its common difference is _____.
- (ii) If the sums of n terms of two arithmetic progressions are in the ratio $(3n + 5) : (5n + 7)$, then their m^{th} terms are in the ratio _____.
- (iii) There are _____ two digit numbers divisible by 7.
- | | (i) | (ii) | (iii) |
|----|-----|-----------------------|-------|
| A. | -11 | $(5m + 1) : (3m + 1)$ | 12 |
| B. | 11 | $(5m + 1) : (3m + 1)$ | 13 |
| C. | -11 | $(3m + 1) : (5m + 1)$ | 13 |
| D. | 11 | $(3m + 1) : (5m + 1)$ | 12 |

48. Which of the following options is incorrect?
- A. Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

- B. If the diagonals of a quadrilateral divide each other proportionally, then it is a trapezium.
- C. If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.
- D. If R is a point on side PQ produced of a parallelogram $PQTU$ and UR intersects QT at S , then $\Delta PUR \sim \Delta STU$.

	(i)	(ii)	(iii)
A.	P	Q	R
B.	R	Q	P
C.	Q	R	P
D.	Q	P	R

49. Match the following and select the correct option.

Column I

Column II

(i) If $\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta} = \frac{81}{49}$, P. $\frac{113}{24}$

then $\frac{\cos \theta + \sin \theta}{\sin \theta - \cos \theta} =$

(ii) $\frac{2}{3}(\cos^4 30^\circ - \sin^4 45^\circ) - 3(\sin^2 60^\circ - \sec^2 45^\circ)$ Q. $\frac{79}{47}$

$+ \frac{1}{4} \cot^2 30^\circ =$

(iii) If $\operatorname{cosec} \theta - \sin \theta = l$ and $\sec \theta - \cos \theta = m$, then $l^2 m^2 (l^2 + m^2 + 3) =$ R. 1

50. Solve the following questions :

- (i) A container is in the form of the frustum of a cone. If its height is 16 cm and the diameter of its lower and upper ends are 16 cm and 40 cm, respectively. Find the cost of the milk that the container can hold, if the cost of milk is ₹ 30 per L. (Take $\pi = 3.14$)
- (ii) A solid is in the form of a cone mounted on a hemisphere in such a way that the centre of the base of the cone just coincide with the centre of the base of the hemisphere. Slant height of the cone is l and r radius of the base of the cone and hemisphere. Find the surface area (in sq. units) of the solid.

	(i)	(ii)
A.	₹ 313.50	$\pi r(2r + l)$
B.	₹ 250.40	$\pi r(2r + l)$
C.	₹ 330.90	$\pi r(r + l)$
D.	₹ 313.45	$\pi r(r + l)$

SPACE FOR ROUGH WORK