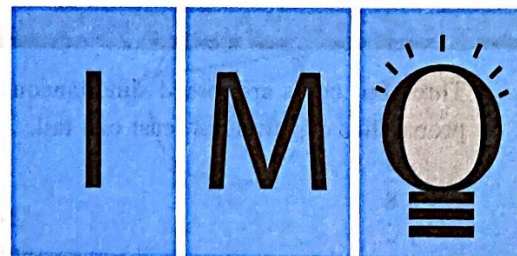


CLASS
9

LEVEL
2



**SOF INTERNATIONAL
MATHEMATICS OLYMPIAD
2019-20**

DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
3. In the school code column in the OMR Sheet, please fill in code allocated to your school and not the exam center code.
4. The Question Paper comprises two sections : **Mathematics** Section (45 Questions) and **Achievers Section** (5 Questions).
Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
5. All questions are compulsory. There is no negative marking. Use of calculator / smart phone is not permitted.
6. There is only ONE correct answer. Choose only ONE option for an answer.
7. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.
Q. 16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is _____.
A. 11.450 kg B. 11.000 kg C. 11.350 kg D. 11.250 kg
As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet.
8. Rough work should be done in the blank space provided in this booklet.
9. Please fill in your personal details in the space provided on this page before attempting the paper.
10. **RETURN THE OMR SHEET AND QUESTION PAPER TO THE INVIGILATOR AT THE END OF THE EXAM.**

16. ● (B) (C) (D)



Inspiring Young Minds Through Knowledge Olympiads

Name:.....

Section:..... SOF Olympiad Roll No.:..... Contact No.:.....

1. Three fair coins are tossed simultaneously. Find the probability of getting at least one tail.
- A. $\frac{3}{8}$
 B. $\frac{7}{8}$
 C. $\frac{1}{2}$
 D. $\frac{1}{4}$

2. A point whose abscissa and ordinate are -3 and -7 respectively, lies in _____.
- A. First quadrant
 B. Second quadrant
 C. Third quadrant
 D. Fourth quadrant

3. Simplify: $(p + q)^3 + (p - q)^3 + 6p(p^2 - q^2)$
- A. $6q^2p$
 B. $8p^3$
 C. $8q^3$
 D. $2pq$

4. If $p = \frac{\sqrt{x+2} + \sqrt{x-2}}{\sqrt{x+2} - \sqrt{x-2}}$, then $x =$
- A. $p + \frac{1}{p}$
 B. $p^2 + 1$
 C. $p - \frac{1}{p}$
 D. $p - 1$

5. Which of the following statements is incorrect?
- A. A solid has 3 dimensions.
 B. A terminated line can be produced indefinitely.
 C. A point has 1 dimension.
 D. The edges of a surface are lines.

6. Find the value of p , if the mean of the following distribution is 7.5.

| | | | | | | |
|-----|---|---|-----|---|----|----|
| x | 3 | 5 | 7 | 9 | 11 | 13 |
| f | 6 | 8 | p | 3 | 8 | 4 |

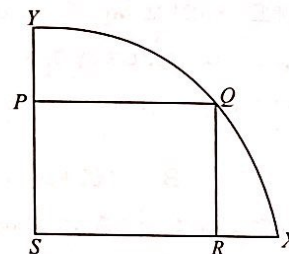
- A. 15
 B. 12
 C. 13
 D. 14

7. The dimensions of a rectangular piece of paper are $28 \text{ cm} \times 16 \text{ cm}$. It is rolled once across the breadth and once across the length to form right circular cylinders of biggest possible surface areas. Find the difference in total surface areas of the two cylinders that will be formed.
- A. 96 cm^2
 B. 84 cm^2
 C. 80 cm^2
 D. 72 cm^2

8. Which of the following is not possible in case of ΔPQR ?
- A. $PQ = 15 \text{ cm}$, $QR = 8 \text{ cm}$, $PR = 17 \text{ cm}$
 B. $\angle P = 40^\circ$, $\angle Q = 60^\circ$, $\angle R = 80^\circ$
 C. $PQ = 6 \text{ cm}$, $QR = 14 \text{ cm}$, $PR = 7 \text{ cm}$
 D. $\angle P = 65^\circ$, $\angle Q = 90^\circ$, $\angle R = 25^\circ$

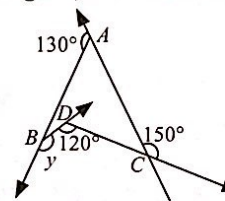
9. The mean of 16 numbers is 8. If 5 is added to every number, then what will be the new mean?
- A. 13
 B. 12
 C. 16
 D. 18

10. In the given figure, $PQRS$ is a rectangle inscribed in a quadrant of a circle. If area of the quadrant is $\frac{3179}{14} \text{ cm}^2$ and $SR = 15 \text{ cm}$, then find the area of the rectangle.



- A. 120 cm^2
 B. 136 cm^2
 C. 140 cm^2
 D. 255 cm^2

11. In the given figure, the value of y is _____.



- A. 140°
 B. 130°
 C. 120°
 D. 145°

12. In ΔPQR , X , Y and Z are the mid-point of the sides PQ , QR and PR respectively. If $ar(\text{quad. } PXYZ) = k ar(\Delta PQR)$, then find the value of k .

- A. $\frac{1}{4}$
 B. $\frac{1}{3}$
 C. $\frac{1}{2}$
 D. $\frac{2}{3}$

13. Factorise : $4\sqrt{3}y^2 + 5y - 2\sqrt{3}$

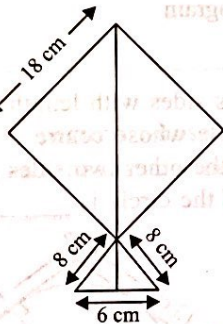
- A. $(\sqrt{3}y - 2)(4y + \sqrt{3})$
 B. $(\sqrt{3}y + 2)(4y - \sqrt{3})$
 C. $(y - 2)(4\sqrt{3}y + 1)$
 D. $(y + 2)(4\sqrt{3}y - 1)$

14. If point $A(p, q)$ lies in the quadrant IV, then

- A. $p < 0, q < 0$
 B. $p < 0, q > 0$
 C. $p > 0, q < 0$
 D. $p > 0, q > 0$



15. A kite is in the shape of a square and an isosceles triangle as shown in the figure. Find the approximate area of the figure.

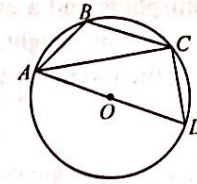


- A. 341.89 cm^2
 B. 346.25 cm^2
 C. 350.45 cm^2
 D. 342.94 cm^2

16. If n is a natural number, then \sqrt{n} is

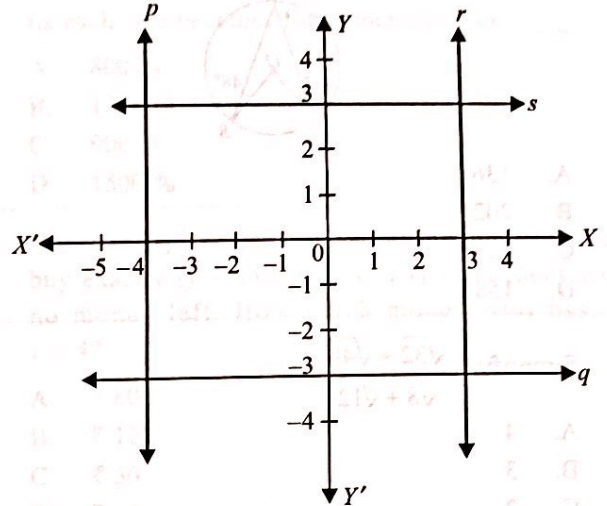
- A. Always a natural number.
 B. Always a rational number.
 C. Always an irrational number.
 D. Sometimes a natural number and sometimes an irrational number.

17. In the given figure, O is the centre of the circle. If $\angle ABC = 130^\circ$, then find $\angle CAD$.



- A. 45°
 B. 50°
 C. 40°
 D. 35°

18. Find the area enclosed between lines p , q , r and s .



- A. 50 sq. units
 B. 42 sq. units
 C. 52 sq. units
 D. 40 sq. units

19. The construction of a triangle ABC with $AB = 8 \text{ cm}$, $\angle B = 45^\circ$ is possible when $BC - AC$ is equal to

- A. 7.5 cm
 B. 8 cm
 C. 9 cm
 D. 8.5 cm

20. If x, y, z are such that $x + y + z = 2$, $x^2 + y^2 + z^2 = 6$, $x^3 + y^3 + z^3 = 8$, then xyz is equal to _____.

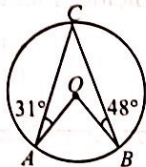
- A. 1
 B. -1
 C. -2
 D. None of these

21. A spherical ball of lead, 3 cm in radius is melted and recast into three spherical balls. The radius of two of these are 1.5 cm and 2 cm respectively. The radius of the third ball is _____.

- A. 2.66 cm
 B. 2.5 cm
 C. 3 cm
 D. 3.5 cm

22. A cone, a hemisphere and a cylinder stand on equal bases and have the same height, the height being equal to the radius of the circular base. Their volumes are in the ratio
- 1 : 2 : 3
 - 3 : 4 : 2
 - 2 : 3 : 4
 - 1 : 2 : 4

23. In the given figure, O is the centre of the circle. Find $\angle AOB$.



- 136°
- 202°
- 128°
- 158°

24. Simplify : $\frac{\sqrt{32} + \sqrt{48}}{\sqrt{8} + \sqrt{12}}$

- 4
- 3
- 2
- 1

25. A tank 3 m long, 2 m wide and 1.5 m deep is dug in a field 22 m long and 14 m wide. If the earth dug-out is evenly spread out over the field, then the approximate rise in the level of the field is _____.

- 0.299 cm
- 1.98 cm
- 2.98 cm
- 4.5 cm

26. How many linear equations are satisfied by $x = -1$ and $y = 4$?

- Only one
- Two
- Three
- Infinitely many

27. Select the incorrect option.

- Every irrational number can be expressed as a non-terminating, non-recurring decimal expansion.
- Every linear polynomial in one variable has a unique solution.
- Every rational number is an integer.
- Every real number is the solution of zero polynomial.

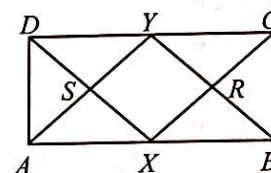
28. If $x^4 + \frac{1}{x^4} = 194$, then find the value of $x^3 + \frac{1}{x^3}$.

- 52
- 56
- 48
- 54

29. A triangle and a parallelogram have a common side and are of equal areas. The triangle having sides 34 cm, 42 cm and 20 cm stands on the parallelogram. The common side of the triangle and the parallelogram is 20 cm. Find the vertical height of the triangle and that of the parallelogram respectively.

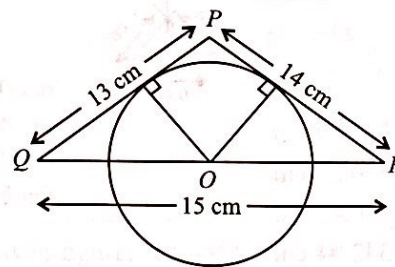
- 33.6 cm, 16.8 cm
- 8.4 cm, 33.6 cm
- 8.4 cm, 16.8 cm
- 33.6 cm, 38.6 cm

30. X, Y are the mid-points of opposite sides AB and DC respectively of a parallelogram $ABCD$. AY and DX are joined intersecting in S ; CX and BY are joined intersecting in R . Then $SXRY$ is a



- Rectangle
- Rhombus
- Parallelogram
- Square

31. A triangle has sides with lengths 13 cm, 14 cm and 15 cm. A circle whose centre O lies on the longest side touches the other two sides as shown in figure. The radius of the circle is _____.



- $\frac{43}{9}$ cm
- $\frac{49}{9}$ cm
- $\frac{56}{9}$ cm
- $\frac{63}{15}$ cm

32. What must be subtracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the resultant is exactly divisible by $2x^2 + x - 1$?

- A. 6
- B. -6
- C. 4
- D. -4

33. Vipul tells Neha, "Sixteen years ago, I was 4 times as old as you were then. Also, eight years from now, I shall be 3 times as old as you will be". If present ages of Neha and Vipul are x and y years respectively, then represent this situation algebraically.

- A. $4x - y = 48; y - 3x = 16$
- B. $x - y = 48; y - 3x = 16$
- C. $x - 4y = 16; y - x = 48$
- D. $5x - y = 48; y - x = 16$

34. If $(2x - 1)$ is a factor of the polynomial $p(x) = 2x^3 + ax^2 + 11x + a + 3$, then find the value of $\frac{5-a}{4}$.

- A. -2
- B. 3
- C. -3
- D. 2

35. If $\alpha + \beta = 10$ and $\alpha^2 + \beta^2 = 58$ ($\alpha > 0$ and $\beta > 0$), then $\alpha^3 + \beta^3 =$ _____.

- A. 320
- B. 370
- C. 280
- D. 480

36. The area of a trapezium whose parallel sides are 9 cm & 16 cm and the distance between these sides is 8 cm, is _____.

- A. 60 cm^2
- B. 72 cm^2
- C. 56 cm^2
- D. 100 cm^2

37. Study the following statements carefully and select the correct option.

Statement-1: The sum of the three altitudes of a triangle is more than the sum of three sides of the triangle.

Statement-2: Any two sides of a triangle are together greater than twice the median drawn to the third side.

- A. Both Statement-1 and Statement-2 are true.
- B. Both Statement-1 and Statement-2 are false.
- C. Statement-1 is true but Statement-2 is false.
- D. Statement-1 is false but Statement-2 is true.

38. P, Q, R are three sets of values of x :

$P: 4, 4, 12, 7, 2, 3, 4$

$Q: 4, 6, 5, 7, 9, 4, 7$

$R: 1, 3, 3, 3, 5$

Which of the following options is correct?

- A. Mean of $R =$ Mode of P
- B. Mean of $P =$ Median of Q
- C. Median of $Q =$ Mode of R
- D. Mean, Median and Mode of R are equal.

39. The percentage increase in the area of a triangle, if its each side becomes four times equal to _____.

- A. 800 %
- B. 1200 %
- C. 900 %
- D. 1500 %

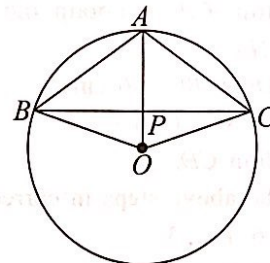
40. Atul has ₹($y^3 + 2ay + b$), with this money he can buy exactly $(y - 1)$ watches or $(y + 1)$ shoes with no money left. How much money Atul has, if $y = 4$?

- A. ₹ 80
- B. ₹ 120
- C. ₹ 30
- D. ₹ 60

41. The graph of $y = -5$ is a line

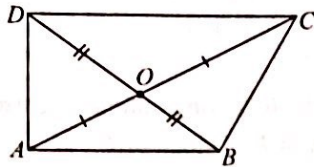
- A. parallel to the y -axis at a distance of 5 units to the left of origin.
- B. parallel to the x -axis at a distance of 5 units above the origin.
- C. parallel to the y -axis at a distance of 5 units to the right of origin.
- D. parallel to the x -axis at a distance of 5 units below the origin.

42. O is the centre of the circle having radius 5 cm. AB and AC are two chords such that $AB = AC = 6$ cm. If OA meets BC at P , then $AP =$ _____.



- A. 4.6 cm
- B. 1.4 cm
- C. 2 cm
- D. None of these

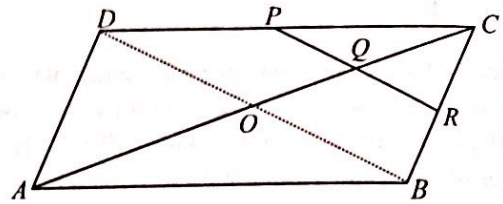
43. In the figure, if $AB + BC + CD + DA < k(AC + BD)$, then find the value of k .



- A. 3
B. 1
C. 2
D. $\frac{1}{2}$
44. If the diagonals of a quadrilateral are perpendicular but not equal, then it is a _____.

- A. Rectangle
B. Square
C. Trapezium
D. Kite

45. In the given figure, $ABCD$ is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. Also, PQ when produced meets BC at R . Then $CR =$ _____.



- A. RB
B. $\frac{1}{3}CB$
C. $\frac{1}{4}CB$
D. None of these

ACHIEVERS SECTION

46. R_1 and R_2 are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $R_1 + 2R_2 = 0$, then find the value of R_1 and R_2 respectively.

- A. 2, -1
B. 4, -2
C. 6, -3
D. 12, -6

47. While constructing a triangle ABC , in which $BC = 6$ cm, $\angle B = 45^\circ$ and $AC - AB = 2$ cm, we follow the following steps :

Step 1 : Draw the perpendicular bisector of CD meeting BD at A .

Step 2 : From ray BX , cut-off line segment BD equal to $AC - AB$ i.e., 2 cm.

Step 3 : Join CA to obtain the required triangle ABC .

Step 4 : Draw $BC = 6$ cm.

Step 5 : Draw $\angle CBX = 45^\circ$.

Step 6 : Join CD .

Arrange the above steps in correct order.

- A. 4, 5, 6, 1, 2, 3
B. 5, 4, 6, 2, 3, 1
C. 4, 5, 2, 6, 1, 3
D. None of these

48. Read the statements carefully and state 'T' for true and 'F' for false.

The internal and external diameters of a hollow cylindrical vessel are 14 cm and 18 cm respectively and its height is 14 cm. Then,

- (i) Its inner curved surface area is 616 cm^2 .
(ii) Its outer total surface area is $\frac{9101}{7} \text{ cm}^2$.
(iii) Its volume is 2154 cm^3 .

- | | | | |
|----|-----|------|-------|
| | (i) | (ii) | (iii) |
| A. | T | T | F |
| B. | T | F | F |
| C. | F | F | F |
| D. | F | T | F |

49. The shirt size worn by a group of 200 persons who bought the shirt from a store is given below.

| Shirt size (in cm) | Number of people |
|--------------------|------------------|
| 36 | 17 |
| 37 | 18 |
| 38 | 25 |
| 39 | 34 |
| 40 | 42 |
| 41 | 32 |
| 42 | 8 |
| 43 | 10 |
| 44 | 14 |

A person is chosen at random, then match Column-I with their corresponding probabilities in Column-II.

| Column - I | Column -II |
|---|------------|
| P. The probability that the person bought shirt of size 42 cm is | 1. $3/10$ |
| Q. The probability that the person bought shirt of size less than 39 cm is | 2. $29/50$ |
| R. The probability that the person bought shirt of size greater than or equal to 41 cm is | 3. $1/25$ |
| S. The probability that the person bought shirt of size less than 43 cm but greater than 38 cm is | 4. $8/25$ |

| | P | Q | R | S |
|----|---|---|---|---|
| A. | 2 | 3 | 4 | 1 |
| B. | 3 | 2 | 4 | 1 |
| C. | 3 | 1 | 4 | 2 |
| D. | 2 | 4 | 3 | 1 |

50. BC is a chord in the circle of centre O . A is a point on arc BC .

(a) If A is on the major arc, then find $\angle BAC + \angle OBC$.

(b) If A is on the minor arc, then find $\angle BAC - \angle OBC$.

| | (a) | (b) |
|----|-------------|------------|
| A. | 120° | 60° |
| B. | 120° | 90° |
| C. | 90° | 60° |
| D. | 90° | 90° |

SPACE FOR ROUGH WORK