IMO

SOF INTERNATIONAL MATHEMATICS OLYMPIAD 2018-19

CLASS 12

QUESTION PAPER SET B

DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Name:..

Section:.............. SOF Olympiad Roll No.:........................ Contact No.:...

Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share with you your marks / result on your mobile number.
3. The Question Paper comprises four sections:

   Logical Reasoning (15 Questions), **Mathematical Reasoning** (20 Questions), **Everyday Mathematics** (10 Questions) and **Achievers Section** (5 Questions)

   Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
4. All questions are compulsory. There is no negative marking. Use of calculator is not permitted.
5. There is only ONE correct answer. Choose only ONE option for an answer.
6. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.

   Q.16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is______.

   A. 11.450 kg   B. 11.000 kg   C. 11.350 kg   D. 11.250 kg

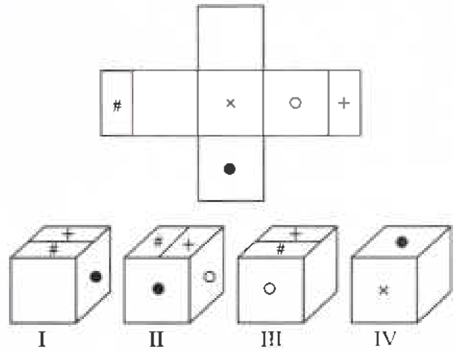
   As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet.   16. ● Ⓑ Ⓒ Ⓓ
7. Rough work should be done in the blank space provided in the booklet.
8. Return the OMR Sheet to the invigilator at the end of the exam.
9. Please fill in your personal details in the space provided on this page before attempting the paper.

SOF

SCIENCE OLYMPIAD FOUNDATION

Inspiring Young Minds Through Knowledge Olympiads

1. A sheet of paper shown in the given figure which has to be folded to form a box. Choose a box from the options, that is similar to the box formed.



A. I and II
B. II, III and IV
C. I, II, III and IV
D. Only IV

2. Find the missing number from the options, if same rule is followed in all the three figures.

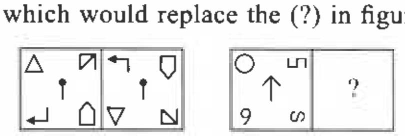
A. 191
B. 263
C. 371
D. None of these

3. If 'A @ B' means 'A is father of B', 'A + B' means 'A is son of B', 'A $ B' means 'A is daughter of B', 'A % B' means 'A is mother of B' and 'A & B' means 'A is husband of B'.
Then, what should come in place of (?) to establish that P is the mother-in-law of T in the given expression ?

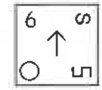
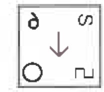
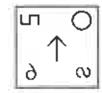
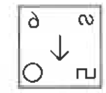
P % Q + R @ S (?) T

A. @
B. Either & or %
C. $
D. &

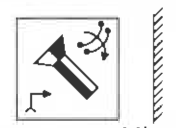
4. There is a certain relationship between figures (1) and (2). Establish a similar relationship between figures (3) and (4) by selecting a suitable figure from the options which would replace the (?) in figure (4).



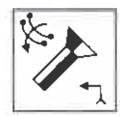
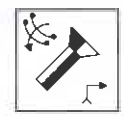
(1) (2) (3) (4)

A. 
B. 
C. 
D. 

5. Select the correct mirror image of the given figure.



Mirror

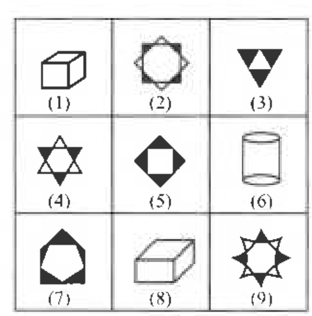
A. 
B. 
C. 
D. 

6. If all the digits in the given arrangement are removed, then how many such symbols are there which are immediately preceded by a letter but not immediately followed by a symbol?

L 3 P A 4 % R 5 @ E J I $ * F 1 U H C # 9 % W I 8 K N T © W 9 A T

A. Two
B. Three
C. Four
D. None of these

7. Group the given figures into three classes on the basis of their identical properties by using each figure only once.



A. 1, 2, 4; 3, 6, 8; 5, 7, 9
B. 1, 6, 7; 2, 4, 9; 3, 5, 8
C. 1, 6, 8; 2, 4, 9; 3, 5, 7
D. 1, 2, 4; 3, 5, 7; 6, 8, 9

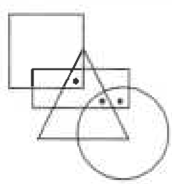
8. If in a certain code language, 'COMPATIBLE' is written as 'CNOMEFKCHU', then how will 'EARTHQUAKE' be written in the same language?

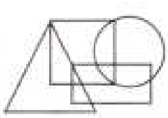
A. RTBJFJRTYG
B. JRTYGFJBTR
C. JRTYGRTBJF
D. EZLQTFJBRT

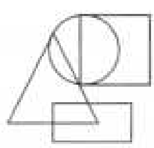
9. Find the number of triangles and squares formed in the given figure respectively.

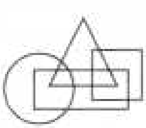
A. 27, 7
B. 29, 5
C. 32, 4
D. None of these

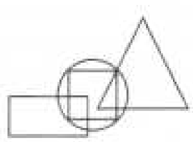
10. Select a figure from the options which satisfies the same conditions of placement of the dots as in the given figure.



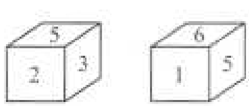
A. 

B. 

C. 

D. 

11. Two positions of a dice are given below. Find the number on the face opposite to the face having the number 3.



A. 1
B. 6
C. 4
D. 5

12. If first and second digits of each of the given numbers are interchanged and one is added to the third digit in each number, then which of the following number will be the second number, when the new numbers are arranged in ascending order?

315 213 768 834 912

A. 315
B. 834
C. 213
D. 912

13. If it is possible to make a meaningful word with the third, fifth, sixth, eighth and twelfth letters of the word PERMUTATIONS (using each letter only once), then which of the following will be the second letter of that word? If no such word can be formed, give 'X' as your answer and if more than one such word can be formed, give 'Y' as your answer.

A. U
B. X
C. T
D. Y

14. Five senior citizens are living in a multi storied building Mr Gupta lives in a flat above Mr Sharma. Mr Patel lives in a flat below Mr Reddy. Mr Sharma lives in a flat above Mr Reddy and Mr Verma lives in a flat below Mr Patel. Who lives in the topmost flat?

A. Mr Patel
B. Mr Reddy
C. Mr Gupta
D. Mr Verma

15. Study the statements carefully.

- M $ N means M is not smaller than N.
- M # N means M is neither smaller than nor equal to N.
- M % N means M is not greater than N.
- M ★ N means M is neither greater than nor equal to N.
- M @ N means M is neither greater than nor smaller than N.

Assuming the following statements to be true, you have to decide which of the following conclusions is/are definitely true?

Statements : P % Q, Q ★ R, S $ R

Conclusions : I. S $ P
II. P ★ S
III. P @ S

A. Only I and II are true
B. Only II and III are true
C. All I, II and III are true
D. None of these

16. The equation of the plane through intersection of planes $x + 2y + 3z = 4$ and $2x + y - z = -5$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$ is

A. $7x - 2y + 3z + 81 = 0$
B. $23x + 14y - 9z + 48 = 0$
C. $51x + 15y - 50z + 173 = 0$
D. None of these

17. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$, $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1 + z_2}{z_1 - z_2}$ may be

A. Real and positive
B. Real and negative
C. Pure imaginary
D. None of these

18. Solution of the differential equation

$$x = 1 + xy\frac{dy}{dx} + \frac{(xy)^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{(xy)^3}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots \text{ is}$$

A. $y = \log_e (x) + C$
B. $y = (\log_e x)^2 + C$
C. $y = \pm\sqrt{(\log_e x)^2 + 2C}$
D. $xy = x^{y} + C$

19. Let p, q, r be three statements. Then $\sim (p \vee (q \wedge r))$ is equal to

A. $(\sim p \wedge \sim q) \wedge (\sim p \wedge \sim r)$
B. $(\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$
C. $(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$
D. $(\sim p \vee \sim q) \vee (\sim p \wedge \sim r)$

20. Let $f(x) = \int_0^x \frac{e^t}{t} dt \ (x > 0)$;

then $e^{-a} [f(x + a) - f(1 + a)] =$

A. $\int_0^x \frac{e^t}{(t + a)} dt$
B. $\int_2^x \frac{e^t}{t + a} dt$
C. $e^{-a} \int_{1+a}^{x+a} \frac{e^t}{t} dt$
D. $\int_0^x \frac{e^{t-a}}{(t + a)} dt$

21. A closed cylindrical can has to be made with 100 m^2 of tin. If its volume is maximum, the ratio of its radius to the height is

A. 1 : 1
B. 1 : 2
C. 2 : 1
D. $\sqrt{2} : 1$

22. The value of x satisfying $\left|\frac{\pi}{6} - \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)\right| < \frac{\pi}{3}$ are

A. $[-2, 2]$
B. $\left[-1, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$
C. $[-1, 1] - \{0\}$
D. None of these

23. The sixth term of an A.P. is equal to 2. The value of the common difference of the A.P. which makes the product $a_1 a_4 a_5$ least is given by

A. $\frac{8}{5}$
B. $\frac{5}{4}$
C. $\frac{2}{3}$
D. None of these

24. Two adjacent sides of a parallelogram $ABCD$ are given by $\overline{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overline{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then $\cos \alpha =$

A. $\frac{8}{9}$
B. $\frac{\sqrt{17}}{9}$
C. $\frac{1}{9}$
D. $\frac{4\sqrt{5}}{9}$

25. The graphs of $f(x) = x^2$ and $g(x) = cx^3$ intersect at two points. If the area of the region bounded between $f(x)$ and $g(x)$ over the interval $[0, 1/c]$ is equal to $\frac{2}{3}$, then the value of $\frac{1}{c} + \frac{1}{c^2}$, is

A. 20
B. 2
C. 6
D. 12

26. If $P(n) = 2 + 4 + 6 + ... + 2n$, $n \in N$, then $P(k) = k(k + 1) + 2 \Rightarrow P(k + 1) = (k + 1)(k + 2) + 2$ for all $k \in N$. So, we can conclude that $P(n) = n(n + 1) + 2$ for all

A. $n \in N$

B. $n > 1, n \in N$

C. $n > 2, n \in N$

D. None of these

27. If the coefficient of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the expansion of $(1 + x)^{14}$ are in A.P., then the value of r, is

A. 5, 9

B. 6, 9

C. 7, 9

D. 2, 3

28. Consider the system of equations in x, y, z as

$x \sin 3\theta - y + z = 0$

$x \cos 2\theta + 4y + 3z = 0$

$2x + 7y + 7z = 0$

If this system has a non-trivial solution, then for any integer n, values of θ are given

A. $\left(n+\frac{(-1)^n}{3}\right)\pi$

B. $\left(n+\frac{(-1)^n}{4}\right)\pi$

C. $\left(n+\frac{(-1)^n}{6}\right)\pi$

D. $\frac{n\pi}{2}$

29. The equations of two sides of a square whose area is 25 square units are $3x - 4y = 0$ and $4x + 3y = 0$. The equations of the other two sides of the square are

A. $3x - 4y \pm 25 = 0$, $4x + 3y \pm 25 = 0$

B. $3x - 4y \pm 25 = 0$, $4x + 3y \pm 5 = 0$

C. $3x - 4y \pm 5 = 0$, $4x + 3y \pm 25 = 0$

D. None of these

30. A number is selected at random from the first twenty-five natural numbers. If it is a composite number, then it is divided by 5. But if it is not a composite number, it is divided by 2. The probability that there will be no remainder in the division is

A. 0.1

B. 0.4

C. 0.2

D. 0.8

31. There are 60 questions in a question paper. If no two students solve the same combination of questions but solve equal number of questions, then the maximum number of students who appeared in the examination, is

A. ${}^{60}C_{29}$

B. ${}^{60}C_{31}$

C. ${}^{60}C_{50}$

D. None of these

32. The maximum value of $z = 2x + 3y$ subject to $3x + 2y \leq 11$, $4x \geq y$, $x \leq 3y$, $x, y \geq 0$ is

A. 7

B. 14

C. 9

D. 18

33. Consider the function $f: R \to (0, 1]$, $f(x) = \sin(\cot^{-1} x)$, then which of the following is/are correct?

A. $f(x)$ is an even function.

B. $f(x)$ is an onto function.

C. Both A and B

D. None of these

34. If $I_n = \begin{vmatrix} 1 & k & k \\ 2n & k^2+k+1 & k^2+k \\ 2n-1 & k^2 & k^2+k+1 \end{vmatrix}$ and $\sum_{n=1}^{k} I_n = 72$, then $k =$

A. 8

B. 9

C. 6

D. 4

35. $\int \frac{e^{\tan^{-1}x}}{1+x^2}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2 + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]dx, (x > 0)$ equals

A. $e^{\tan^{-1}x} \cdot \tan^{-1}x + C$

B. $\frac{e^{\tan^{-1}x}(\tan^{-1}x)^2}{2} + C$

C. $e^{\tan^{-1}x}\left(\sec^{-1}\sqrt{1+x^2}\right)^2 + C$

D. $e^{\tan^{-1}x}\left(\operatorname{cosec}^{-1}\left(\sqrt{1+x^2}\right)\right)^2 + C$

36. In a city no persons have identical set of teeth and there is no person without a tooth. Also no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, then the maximum population of the city is

A. 2^{32}

B. $2^{32}-1$

C. $2^{32}-2$

D. $2^{32}-3$

37. The cost of an apple is twice as that of a banana and the cost of a banana is 25% less than that of a guava. If the cost of each type of fruit is increased by 10%, then the % change in the cost of 4 bananas, 2 apples and 3 guavas is

A. 10% decrease

B. 10% increase

C. 12% decrease

D. 12% increase

38. Two vertical lamp-posts of equal height stands on either side of a road 50 m wide. At a point P on the road between them, the elevation of the tops of the lamp-posts are 60° and 30°. Find the distance of P from the lamp post which makes angle of 60°.

A. 25 m

B. 12.5 m

C. 16.5 m

D. 20.5 m

39. Three athletes A, B and C participate in a race. Both A and B have the same probability of winning the race and each is twice as likely to win as C. The probability that B or C wins the race is

A. $\frac{2}{3}$

B. $\frac{3}{5}$

C. $\frac{3}{4}$

D. None of these

40. The number of oranges in three baskets are in the ratio of 3 : 4 : 5. In which ratio the number of oranges in first two baskets must be increased so that the new ratio becomes 5 : 4 : 3?

A. 1 : 3

B. 2 : 1

C. 3 : 4

D. 2 : 3

41. A boat takes 6 hours to travel from place M to N downstream and back from N to M upstream. If the speed of the boat in still water is 4 km/hr, what is the distance between the two places?

A. 8 km

B. 12 km

C. 6 km

D. Data inadequate

42. Vijay and Vinay are working on an assignment, Vijay takes 6 hours to type 45 pages on a computer, while Vinay takes 4 hours to type 42 pages. How much time will they take, working together on two different computers to type an assignment of 144 pages?

A. 9 hours

B. 7 hours

C. 8 hours

D. $8\frac{1}{2}$ hours

43. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest on ₹ 12000 after 3 years at the same rate of interest?

A. ₹ 2160

B. ₹ 3972

C. ₹ 3120

D. ₹ 6240

44. A and B are partners in a business. A contributes $\frac{1}{4}$ of the capital for 15 months and B reduced to $\frac{2}{3}$ of the profit. Find for how long B's money was used?

A. 10 months

B. 12 months

C. 8 months

D. 6 months

45. There are 8 pipes attached with a tank out of which some are inlets and some are outlets. Every inlet pipe can fill the tank in 12 hours and every outlet pipe can empty the tank in 16 hours. When all the pipes are working together, the tank is filled up in $4\frac{4}{11}$ hours. If pair (a, b) denotes the number of inlets and outlets pipe respectively then which of the following is correct option?

A. (6, 2)

B. (4, 4)

C. (5, 3)

D. (3, 5)

46. Read the statements carefully and select the correct option.

Statement-I : If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then points having position vectors $x_1\vec{a}+y_1\vec{b}, x_2\vec{a}+y_2\vec{b}$ and $x_3\vec{a}+y_3\vec{b}$ are collinear if $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Statement-II : Three points with position vectors $\vec{a},\vec{b},\vec{c}$ are collinear iff there exist scalars x, y, z not all zero such that $x\vec{a}+y\vec{b}+z\vec{c}=\vec{0}$, where $x+y+z=0$

A. Both Statement-I and Statement-II are true.
B. Statement-I is true but Statement-II is false.
C. Statement-I is false but Statement-II is true.
D. Both Statement-I and Statement-II are false.

47. Answer the following questions.

(i) Find the area of the region bounded by the curve $y = x(4 - x)$, the x-axis and the lines $x = 0$, $x = 5$.

(ii) Find the area of the region bounded by $y = x^2$ and $y = |x|$.

	(i)	(ii)
A.	25 sq. units	$\frac{3}{2}$ sq. units
B.	13 sq. units	$\frac{1}{3}$ sq. units
C.	18 sq. units	$\frac{2}{3}$ sq. units
D.	12 sq. units	$\frac{1}{3}$ sq. units

48. Match the following and select the correct option.

Column-I	Column-II
(i) The set of all points where $f(x)=\sqrt[3]{x^2\lvert x\rvert}$ is differentiable is	(p) $(0, \infty)$
(ii) If $f(x)=\begin{cases} x^m \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x=0 \end{cases}$ is continuous at $x = 0$, then $m \in$	(q) $\{-1, 0, 1\}$
(iii) Let $f(x) = \max\{x, x^3\}$, $x \in R$. The set of points where $f(x)$ is not differentiable is	(r) $(-\infty, 0) \cup (0, \infty)$

A. (i) → (r); (ii) → (q); (iii) → (p)
B. (i) → (q); (ii) → (p); (iii) → (r)
C. (i) → (r); (ii) → (p); (iii) → (q)
D. (i) → (p); (ii) → (r); (iii) → (q)

49. Read the given statements carefully and state 'T' for true and 'F' for false.

(i) If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}, \Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$, then $\Delta_1 + \Delta_2 = 0$.

(ii) If ω is an imaginary cube root of unity, then the value of $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix}$ is equal to $-3\omega^2$.

(iii) The value of determinant $\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$ is independent of β.

	(i)	(ii)	(iii)
A.	T	T	T
B.	T	F	T
C.	T	T	F
D.	F	F	T

50. Fill in the blanks and select the correct option.

(i) The angle between the lines whose direction numbers are <1, 2, 2> and <3, 2, –6> is ________.

(ii) The values of p and q for which the line joining points (3, 2, 5) and (p, 5, 0) be parallel to the line joining points (1, 3, q) and (6, 4, – 1) are ________ and ________ respectively.

(iii) The direction cosines of line passing through two points (–2, 4, –5) and (1, 2, 3) are ________.

	(i)	(ii)	(iii)
A.	$\sin^{-1}\left(\frac{4}{19}\right)$	$18, \frac{3}{4}$	$<\frac{2}{\sqrt{77}}, \frac{-3}{\sqrt{77}}, \frac{8}{\sqrt{77}}>$
B.	$\sin^{-1}\left(\frac{5}{21}\right)$	$12, \frac{3}{4}$	<3, –2, 8>
C.	$\cos^{-1}\left(\frac{5}{21}\right)$	$18, \frac{2}{3}$	$<\frac{3}{\sqrt{77}}, -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}>$
D.	$\cos^{-1}\left(\frac{5}{21}\right)$	$16, \frac{3}{5}$	<3, –2, 8>

SPACE FOR ROUGH WORK

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