IMO

SOF INTERNATIONAL MATHEMATICS OLYMPIAD
2018-19

CLASS 11

QUESTION PAPER SET **B**
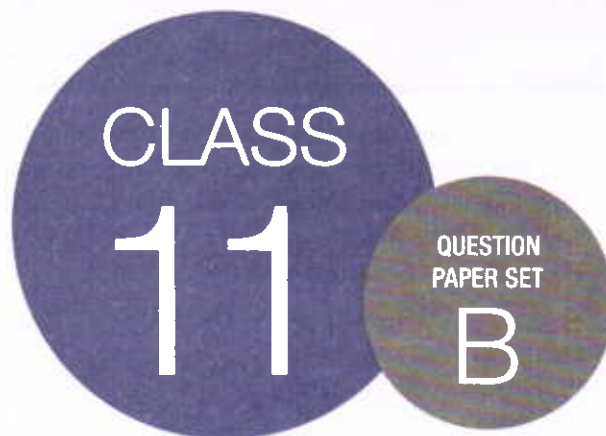
DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Name:..

Section:.............. SOF Olympiad Roll No.:........................... Contact No.:..

Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.

2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share with you your marks / result on your mobile number.

3. The Question Paper comprises four sections:

   Logical Reasoning (15 Questions), **Mathematical Reasoning** (20 Questions), **Everyday Mathematics** (10 Questions) and **Achievers Section** (5 Questions)

   Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.

4. All questions are compulsory. There is no negative marking. Use of calculator is not permitted.

5. There is only ONE correct answer. Choose only ONE option for an answer.

6. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.

   Q.16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is_____.

   A. 11.450 kg   B. 11.000 kg   C. 11.350 kg   D. 11.250 kg

   As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet.   16. ● ⒝ ⒞ ⒟

7. Rough work should be done in the blank space provided in the booklet.

8. Return the OMR Sheet to the invigilator at the end of the exam.

9. Please fill in your personal details in the space provided on this page before attempting the paper.

1. If the positions of the first and last letters in each of the given words are interchanged and then the words thus formed (not necessarily meaningful) are arranged alphabetically from left to right, then which of the following will be second from the left ?

    PIN ROD DEN SAT FUN

    A. SAT
    B. DEN
    C. ROD
    D. PIN

2. A cube has six faces each of a different colour. The red face is opposite to black. The green face is in between red and black. The blue face is adjacent to white face and brown face is adjacent to blue face. The four colours adjacent to green are

    A. Red, black, brown and white
    B. Red, black, brown and blue
    C. Red, black, blue and white
    D. Red, brown, blue and white

3. Which of the following figures will complete the given figure matrix?
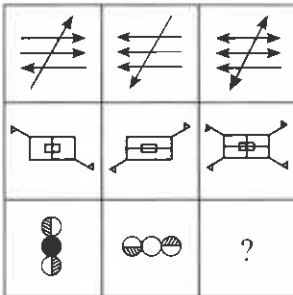
4. Two rows of numbers are given. The resultant number in each row is to be worked out separately based on the following rules. The operations on numbers progress from left to right.

    Rules:
    (i) If an odd number is followed by another odd number which is a perfect square, they are to be added.
    (ii) If an even number is followed by an odd number or an odd number is followed by an even number, they are to be multiplied.

    (iii) If an even number is followed by another even number, the first number is to be divided by the second number.
    (iv) If an even number which is a multiple of 5 is followed by an odd number which is also a multiple of 5, then the second number is to be subtracted from the first number.

    24   3   18   2
    17   81   14   8

    What is the product of the resultants of the two rows?
    A. 108
    B. 112
    C. 98
    D. None of these

5. Some symbols and digits are coded as follows.

Digit/Symbol	1	≠	S	9	8	6	%	@	©	7	2	4	*	3	5
Letter Code	B	D	E	N	I	V	R	G	H	K	T	J	P	F	A

While coding the given combination of symbols and digits, following conditions are also to be observed.
    I. If both the first and the last elements in the group are odd digits, then both are to be coded as 'Y'.
    II. If the first element is a symbol and the last element is an even digit, the codes for the first and the last elements are to be interchanged.
    III. If the first element is an odd digit and the last element is a symbol, both are to be coded as 'Z'.
    IV. If the first element is an even digit and the last element is an odd digit, both are to be coded as the code for the odd digit.

    ©8143*$

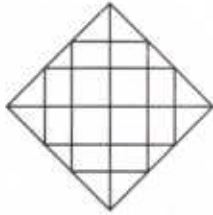   A. EIBJFPH
    B. VIBJFPY
    C. EIBJFP#
    D. HIBJFPE

6. Which of the following options will continue the same series as established by the Problem Figures?

Problem Figures
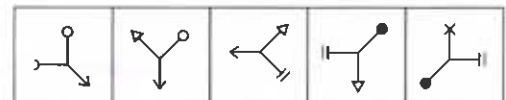
7. Find the number of triangles formed in the given figure.

A. 16
B. 20
C. 27
D. None of these

8. P, T, V, R, M, D, K and W are sitting around a circular table facing the centre. V is second to the left of T. T is fourth to the right of M. D and P are not immediate neighbours of T. D is third to the right of P. W is not an immediate neighbour of P. P is to the immediate left of K. What is R's position with respect to V?

A. Third to the right
B. Fifth to the right
C. Third to the left
D. Second to the left

9. A transparent sheet with a pattern and a dotted line on it is given. Select a figure from the options as to how the pattern would appear when the transparent sheet is folded along the dotted line.
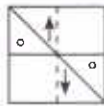

10. Two buses starts from opposite points on a main road 120 km apart. The first bus A runs for 30 km along the main road and takes a right turn and then runs for 10 km. It then turns left and run for another 30 km and takes the direction back to reach the main road. In the meantime due to minor breakdown, the other bus B has run only 40 km along the main road. What would be the distance between two buses at this point?
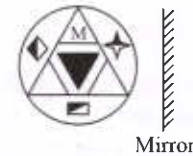
A. 10 km
B. 30 km
C. 40 km
D. None of these

11. If 'a' stands for '−', 'b' stands for '+', 'g' stands for '×' and 'd' stands for '÷', then which of the following is incorrect?

A. $137a\ 56a\ 3g\ 4b\ 5a\ 8g8 = -2$
B. $56d\ 28g\ 19a\ 8a\ 15b3 = 20$
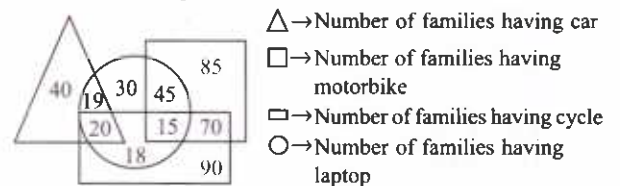C. $23a\ 14b\ 15g\ 9d\ 9 = -6$
D. All of these

12. Find the odd one out.
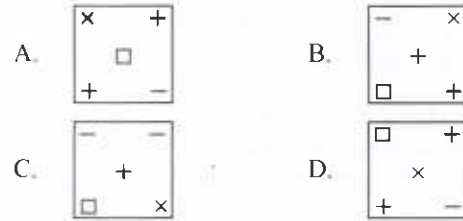
13. Select the correct mirror image of the given figure.

14. Study the given Venn diagram carefully and answer the following question.

△→Number of families having car
□→Number of families having motorbike
▱→Number of families having cycle
O→Number of families having laptop

Which number represents the families who have motorbike and cycle both but do not have car or laptop?

A. 30
B. 70
C. 20
D. Can't be determined

15. Read the following information carefully to answer the question that follows.

(i) P, Q, R, S, T and U are the six members of a family.
(ii) U is the granddaughter of T.
(iii) S is the grandmother of P.
(iv) R is mother of U and wife of Q.
(v) R's mother is S.
(vi) There are two married couples in the family.
What is R's relation with P?

A. Mother
B. Grandmother
C. Daughter
D. Cannot be determined

16. Which of the following options is always true?
    A. $(p \to q) \cong (\sim q \to \sim p)$
    B. $\sim (p \vee q) \cong (\sim p \vee \sim q)$
    C. $\sim (p \to q) \cong (p \vee \sim q)$
    D. $\sim (p \wedge q) \cong (\sim p \wedge \sim q)$

17. AB is a chord of the parabola $y^2 = 4ax$ with vertex at A, BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the x-axis is
    A. a
    B. $2a$
    C. $8a$
    D. None of these

18. The sum of the first n terms of the arithmetic progression is equal to half the sum of the next n terms of the same progression. Find the ratio of the sum of the first $3n$ terms of the progression to the sum of its first n terms.
    A. $5 : 2$
    B. $6 : 1$
    C. $7 : 3$
    D. $8 : 5$

19. If the function $g(x)$ is defined by
    $$g(x) = \frac{x^{200}}{200} + \frac{x^{199}}{199} + \frac{x^{198}}{198} + \ldots + \frac{x^2}{2} + x + 5, \text{ then}$$
    $g'(0) =$
    A. 200
    B. 5
    C. 1
    D. 100

20. A drawer contains 5 brown socks and 4 blue socks well mixed. A man reaches the drawer and pulls out 2 socks at random. What is the probability that they match?
    A. $\dfrac{4}{9}$
    B. $\dfrac{5}{8}$
    C. $\dfrac{5}{9}$
    D. $\dfrac{7}{12}$

21. The solution set of the equation
    $\sqrt{3x^2 - 7x - 30} - \sqrt{2x^2 - 7x - 5} = x - 5$ is
    A. $\left\{6, -\dfrac{5}{2}\right\}$
    B. $\left\{6, -\dfrac{5}{2}, 5\right\}$

C. $\{6, 5\}$
D. None of these

22. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of
    $$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots + \left(z^6 + \frac{1}{z^6}\right)^2 \text{ is}$$
    A. 54
    B. 6
    C. 12
    D. 18

23. If $a = \log 2$, $b = \log 3$, $c = \log 7$ and $6^x = 7^{x+4}$, then $x =$
    A. $\dfrac{4b}{c+a-b}$
    B. $\dfrac{4c}{a+b-c}$
    C. $\dfrac{4b}{c-a-b}$
    D. $\dfrac{4a}{a+b-c}$

24. A chord with midpoint $\left(3, \dfrac{1}{2}\right)$ is drawn to the ellipse $x^2 - 4x + 4y^2 = 0$. The distance of the origin from it is
    A. $\dfrac{3}{\sqrt{5}}$
    B. $\dfrac{2}{\sqrt{5}}$
    C. $\dfrac{4}{\sqrt{5}}$
    D. None of these

25. The equation $\sin^4 x - 2\cos^2 x + a^2 = 0$ is solvable for $x \in R$, if
    A. $-\sqrt{2} \le a \le \sqrt{2}$
    B. $-\sqrt{3} \le a \le \sqrt{3}$
    C. $-1 \le a \le 1$
    D. None of these

26. Range of the function $f(x) = \sqrt{\dfrac{x}{1+x}}$ is
    A. $(0, \infty)$
    B. $[0, \infty)$
    C. $[0, \infty) - \{1\}$
    D. None of these

27. The mean of the values 0, 1, 2, 3, ..., n with the corresponding weights $^nC_0, ^nC_1, ..., ^nC_n$ respectively, is

    A. $\dfrac{n+1}{2}$ B. $\dfrac{n}{2}$

    C. $\dfrac{2^n-1}{2}$ D. $\dfrac{2^n+1}{2}$

28. The equation $\sqrt{\{(x-2)^2+y^2\}}+\sqrt{\{(x+2)^2+y^2\}}=4$ represents

    A. A circle
    B. A pair of lines
    C. A parabola
    D. A line segment $y = 0, -2 < x < 2$

29. In a group of boys, two boys are brothers and in this group 6 more boys are there. In how many ways can they sit if the brothers are not to sit along with each other?

    A. $2 \times 6!$
    B. $^7P_2 \times 6!$
    C. $^7C_2 \times 6!$
    D. None of these

30. The points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are

    A. Collinear
    B. Vertices of a rectangle
    C. Vertices of a parallelogram
    D. Vertices of a square

31. The solution of
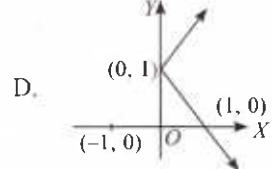$\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[6]{3}} x + ... + \log_{\sqrt[16]{3}} x = 36$ is
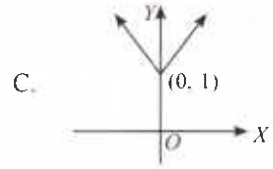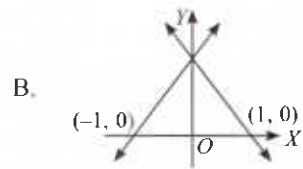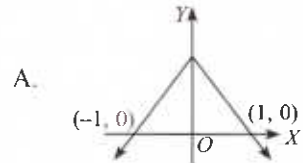
    A. 3
    B. $4\sqrt{3}$
    C. 9
    D. $\sqrt{3}$

32. Find the coordinates of the circumcentre of the triangle whose vertices are (8, 6), (8, –2) and (2, –2).

    A. (2, 3)
    B. (1, 4)
    C. (5, 2)
    D. (0, 5)

33. Which one of the following graphs represents the function $y = 1 + |x|$ for all $x \in R$?

    A.

    B.

    C.

    D.

34. The statement
$(x + a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a^1 + ... + {}^nC_r x^{n-r} a^r + ... + {}^nC_n a^n$ is true for

    A. All rational numbers n
    B. For all natural numbers n
    C. For all real numbers n
    D. None of these

35. If sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, then find the value of k.

    A. 10
    B. 12
    C. 18
    D. None of these

EVERYDAY MATHEMATICS

36. Two pipes A and B can fill a tank in 24 mins and 32 mins respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 mins?

    A. 16 mins
    B. 20 mins
    C. 8 mins
    D. 15 mins

37. A car can hold 2 persons in the front seat and 1 in the rear seat. If among 6 persons only 2 can drive, the number of ways, in which the car can be filled, is

    A. 10
    B. 18
    C. 40
    D. None of these

38. A man buys 50 kg of oil at ₹10 per kilogram and another 40 kg of oil at ₹12 per kilogram and mixes them. He sells the mixture at the rate of ₹11 per kilogram. What will be his gain percent, if he is able to sell the whole lot?

   A. $1\dfrac{1}{49}\%$

   B. $100\left(\dfrac{10}{49}\right)\%$

   C. $10\left(\dfrac{1}{49}\right)\%$

   D. None of these

39. In a college, 20% students fail in Mathematics, 25% in Physics and 12% in both subjects. A student of this college is selected at random. The probability that the selected student is failed in both Mathematics and Physics, is

   A. $\dfrac{1}{20}$

   B. $\dfrac{3}{25}$

   C. $\dfrac{12}{25}$

   D. $\dfrac{3}{5}$

40. A machine depreciates its value each year at the rate of 10% of its previous value. However, every second year there is some maintenance work so that in that particular year, depreciation is only 5% of its previous value. If at the end of the fourth year, the value of the machine stands at ₹146205, then find the value of machine at the start of the first year.

   A. ₹190000
   B. ₹200000
   C. ₹195000
   D. ₹210000

41. Two trains starting at the same time from two stations 200 km apart and going in opposite directions cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?

   A. 9 : 20
   B. 11 : 9
   C. 11 : 20
   D. None of these

42. The horizontal distance between two trees of different heights is 60 m. The angle of depression of the top of the first tree, when seen from the top of the second tree is 45°. If the height of the second tree is 80 m, find the height of the first tree.

   A. 140 m
   B. 20 m
   C. $100\sqrt{3}$ m
   D. $200\sqrt{3}$ m

43. A cylindrical container is filled with ice-cream, whose diameter is 12 cm and height is 15 cm. The whole ice-cream is distributed to 10 children in equal cones having hemispherical tops. If the height of conical portion is twice the diameter of its base, find the diameter of the ice-cream cone.

   A. 5 cm
   B. 8 cm
   C. 12 cm
   D. 6 cm

44. If a team of four persons is to be selected from 8 males and 8 females, then in how many ways can the selections be made to include at least one male?

   A. 3500
   B. 875
   C. 1200
   D. 1750

45. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy gets (in ₹) if he starts working on the 1st of February and finishes on the 20th of February?

   A. 2^{20}
   B. $2^{20} - 1$
   C. $2^{19} - 1$
   D. 2^{19}

ACHIEVERS SECTION

46. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

The standard deviation and mean diameter of the circles respectively are

   A. 5.55, 44.5
   B. 5.25, 43.5
   C. 5.55, 43.5
   D. 5.25, 44.5

47. Read the given statements carefully and select the correct option.

Statement-I : For every natural number $n \geq 2$,
$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}} > \sqrt{n}.$$
Statement-II : For every natural number $n \geq 2$,
$\sqrt{n(n+1)} < n+1$.

  A.   Both Statement-I and Statement-II are true.
  B.   Statement-I is true but Statement-II is false.
  C.   Statement-I is false but Statement-II is true.
  D.   Both Statement-I and Statement-II are false.

48. Match the following :

Column-I		Column-II
P. $\ ^{22}C_0 + \ ^{22}C_2 + \ ^{22}C_4 + ... + \ ^{22}C_{10} =$	(1)	$\frac{1}{2}\ ^{20}C_{10}$
Q. $\ ^{20}C_0 - \ ^{20}C_1 + \ ^{20}C_2 - \ ^{20}C_3 + ... + \ ^{20}C_{10} =$	(2)	5×2^{20}
R. $\sum_{r=0}^{10} r \cdot \ ^{20}C_r$	(3)	2^{20}

  A.   P→(2), Q→(1), R→(3)
  B.   P→(2), Q→(3), R→(1)
  C.   P→(3), Q→(1), R→(2)
  D.   P→(1), Q→(3), R→(2)

49. Read the statements carefully and state 'T' for true and 'F' for false.

(i)   Number of words with or without meaning, that can be formed using all the letters of the word EQUATION, using each letter exactly once are, 40320.

(ii)   The number of permutations of n different things taken r at a time, when repetition is allowed, is $\frac{n!}{(n-r)!}$.

(iii)   The number of ways in which 4 red, 3 yellow and 2 green discs can be arranged in a row, if the discs of same colour are identical, are 1260.

	(i)	(ii)	(iii)
A.	F	T	F
B.	T	F	T
C.	T	T	F
D.	F	F	T

50. Fill in the blanks and select the correct option.

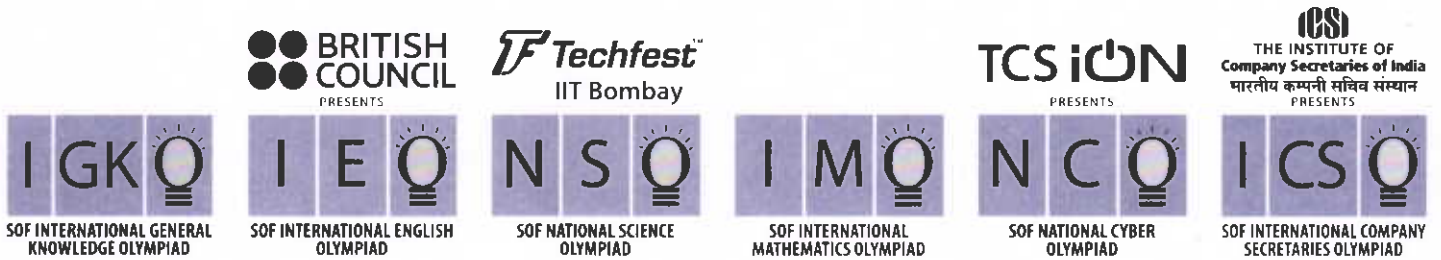(i) $4 \cos 18° - 3 \sec 18° - 2 \tan 18° = $ __(P)__ .

(ii)   If $\tan A = \frac{m}{m-1}$ and $\tan B = \frac{1}{2m-1}$, then $A - B = $ __(Q)__ .

(iii)   If $\cos^3 \theta \sin 3\theta + \sin^3 \theta \cos 3\theta = m \sin 4\theta$, then $m = $ __(R)__ .

	(P)	(Q)	(R)
A.	$\sqrt{2}$	$\frac{\pi}{3}$	2
B.	0	$\frac{\pi}{4}$	3
C.	0	$\frac{\pi}{4}$	$\frac{3}{4}$
D.	4	π	$\frac{1}{2}$

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK