



INDIAN SCHOOL AL WADI AL KABIR
PO 513, PC 117, WADI KABIR, SULTANATE OF OMAN
Department of Mathematics, 2018-2019

SUMMER MATHEMATICS HOLIDAY HOMEWORK [2018-19] CLASS-XII

Q.No Questions

1 or 2 marks

1. Find the value of x and y if: $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

2. Find the value of $\tan(\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3})$

3. Evaluate: $\int \frac{1 + \cot x}{x + \log \sin x} dx$

4. If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$.

5. Find: $\int e^x \frac{\sqrt{1 + \sin 2x}}{1 + \cos 2x} dx$

4 marks

6. Find a matrix X such that $2A + B + X = 0$ where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

7. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find a and b .

8. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{1a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{1a}{b}\right) = \frac{2b}{a}$.

9. Evaluate: $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$
10. If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{d^2y}{dx^2} = 0$.
11. If $y = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right)$, then show that $\frac{dy}{dx} = \sqrt{a^2 - x^2}$.
12. Evaluate: $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$
13. If $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right) = \alpha$ then prove that $x^2 = \sin^2 \alpha$.
14. If $\cos^{-1} \left(\frac{x}{a} \right) + \cos^{-1} \left(\frac{y}{b} \right) = \alpha$ prove that $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$
15. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, then prove that $\sin y = \tan^2 \left(\frac{x}{2} \right)$
16. Evaluate: $\int_{-1}^2 (7x - 5) dx$, as a limit of sums.

6 marks

17. Evaluate $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$
18. If the function $f : R \rightarrow R$ be defined by $f(x) = 2x - 3$ and $g(x) = x^2 + 5$, then find $f \circ g$ and show that $f \circ g$ is invertible. Also find $(f \circ g)^{-1}$, hence find $(f \circ g)^{-1}(9)$.
19. Using elementary transformations, find the inverse of the following matrix:
- $$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

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Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

Answers: 1) $x = 3, y = 3$ 2) $\frac{17}{6}$ 3) $\log|x + \log \sin x| + C$ 4) $(A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

5) $\frac{1}{2} e^x \sec x + c$ 6) $\begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$ 7) $a=1, b=4$ 9) $\frac{5}{2\pi} - \frac{1}{\pi^2}$

12) $I = \frac{\pi}{4}$ 16) $-\frac{9}{2}$ 17) $\frac{\pi^2}{2ab}$

18) $(f \circ g)^{-1} : R \rightarrow R$ defined by $(f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-7}{2}}$

19) $A^{-1} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$

20) $\frac{1}{20} \log 3$

Submission Date: 7th Aug 2018

All the Best!